SYSTEMS OF LINEAR INEQUALITIES

An inequality is generally used when making statements involving terms such as at most, at least, between, greater than, or less than. These statements are inequality statements. An inequality is a statement that two expressions may or may not be equal. Based on the sign of inequality used, one expression may be larger than another. The tables below show inequality signs and their meanings.

<table>
<thead>
<tr>
<th>Sign</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;</td>
<td>Greater than</td>
</tr>
<tr>
<td>&lt;</td>
<td>Less than</td>
</tr>
<tr>
<td>≥</td>
<td>Greater than or equal to</td>
</tr>
<tr>
<td>≤</td>
<td>Less than or equal to</td>
</tr>
<tr>
<td>≠</td>
<td>Unequal</td>
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</tbody>
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GRAPHING SYSTEMS OF LINEAR INEQUALITIES

Systems of linear inequalities are usually solved graphically. Here is a brief review of graphing a linear inequality.

- First: Graph the inequality as if it were an equation.
- Next: Connect the data points with a dashed line or a solid line, depending on the sign of inequality.
  For ≥ or ≤, use a solid line to show that points on the line are part of the solution.
  For > or <, use a dashed line to show that points on the line are not part of the solution.
- Finally: Shade the area in which the solutions lie: the area above or below or to the right or left of the line. To find the area where the solutions lie, choose a test point not too near the line. Substitute the coordinates of that chosen point in the inequality. If the result is true, shade the area on
EXAMPLE:

You could choose (5, 0) as a test point:
\[ y \leq 2x + 3 \]
\[ 0 \leq 2(5) + 3 \]
\[ 0 \leq 13 \]
This is true.
Shade on the side that includes point (5, 0).

If the result had been false, you would shade on the other side of the line, the side that does not include (5,0)

TRY THESE

For each of the following systems, graph the system and label the solution A

1. \[ \begin{align*}
3x - 2y &> 1 \\
2x + 3y &\leq 18
\end{align*} \]

2. \[ \begin{align*}
2x - 3y &\leq 2 \\
x + y &\leq 6
\end{align*} \]
3. \[ 3x - y \geq -7 \]
\[ 3x + y \leq -2 \]

4. The total weight of item A and B is greater than or equal to 3 pounds but less than or equal to 15 pounds. Using a system of inequalities, graph the solution.
1. Brenda’s high school theater can seat at most 400 people. Adult tickets are $5 and student tickets are $2. The school must make at least $1,000 for the show to go on. Which region represents the possible numbers of adult and student tickets sold that meet the given conditions?

   a. Region A  
   b. Region B  
   c. Region C  
   d. Region D

2. The owner of a restaurant determines she can spend no more than $700 to buy coffee for the next month. At wholesale prices, the regular coffee she uses costs $3.00 per pound and the decaffeinated coffee costs $3.50 per pound. The owner estimates she will need at least 65 pounds of coffee for the month. Which graph represents the possible combinations of the number of pounds of regular coffee, x, and the number of pounds of decaffeinated coffee, y, that meet these conditions?

   [F]  
   [G]  
   [H]  
   [I]
3. The owner of a restaurant determines she can spend no more than $1500 to buy coffee for the next month. At wholesale prices, the regular coffee she uses costs $4.00 per pound and the decaffeinated coffee costs $5.00 per pound. The owner estimates she will need at least 75 pounds of coffee for the month. Which graph represents the possible combinations of the number of pounds of regular coffee, \( x \), and the number of pounds of decaffeinated coffee, \( y \), that meet these conditions?

(F) 

(G) 

(H) 

(I) 

4. A college student decides that school work limits him to a total of no more than 33 hours per week at his two part-time jobs. He earns $11 per hour hanging wall paper and he has a sales job that pays $7 per hour. He needs to earn at least $310 per week to cover his expenses. Write a system of inequalities that shows the various numbers of hours he can work at each job. Let \( h \) represent the number of hours spent hanging wall paper and let \( s \) represent the number of hours spent working at the sales job.
5. A college student decides that school work limits him to a total of no more than 36 hours per week at his two part-time jobs. He earns $6 per hour hanging wall paper and he has a sales job that pays $8 per hour. He needs to earn at least $245 per week to cover his expenses. Write a system of inequalities that shows the various numbers of hours he can work at each job. Let \( h \) represent the number of hours spent hanging wall paper and let \( s \) represent the number of hours spent working at the sales job.

6. Members of a school boosters club want to sell at least 10 school jackets and at least 21 caps during a fundraiser to cover their regular club expenses. The club will make $14 for every jacket sold and $2 for every cap sold. The club also wants the total amount of money earned to be at least $300.

**Part A** Write a system of inequalities that shows how many jackets and caps the club members need to sell to meet the conditions described above. Let \( x \) represent the number of school jackets sold and let \( y \) represent the number of caps sold.

**Part B** Graph the system of inequalities from Part A, shading in the region that represents the possible solutions to the system of equations. Locate and label the point \((14, 35)\) on the graph.

**Part C** Will the club meet its goal of earning $300 if it sells 14 jackets and 35 caps? Explain how you know. How much above or below its goal will the club be?