In this chapter, we solve systems of linear equations in two and three variables. One of the skills we need is to find the point of intersection between two lines.

**Are You Prepared?**

To practice, graph the five lines on the grid provided. Then answer the questions and place the letter corresponding to each answer in the space below. This will define an important vocabulary word for this chapter.

1. Line 1. \( y = x + 2 \)
2. Line 2. \( y = -x + 4 \)
3. Line 3. \( x - y = 1 \)
4. Line 4. \( y = -2 \)
5. Line 5. \( x + 4 = 1 \)

1. The point of intersection between lines 1 and 2. \( T (-1, -2) \)
2. The point of intersection between lines 3 and 4. \( I (-3, -1) \)
3. The point of intersection between lines 5 and 1. \( U \) Line 2
4. The line that is parallel to line 3. \( O (1, 3) \)
5. The line that is vertical. \( S \) Line 1
6. The line that has (3, 1) as a solution. \( N \) no point of intersection
7. The point of intersection between lines 1 and 3. \( L \) Line 5

A ___ ___ ___ ___ ___ ___ ___ ___ to a system of linear equations is a point of intersection between two lines.
1. Solutions to Systems of Linear Equations

A linear equation in two variables has an infinite number of solutions that form a line in a rectangular coordinate system. Two or more linear equations form a system of linear equations. For example:

\[
\begin{align*}
  x - 3y &= -5 \\
  2x + 4y &= 10
\end{align*}
\]

A solution to a system of linear equations is an ordered pair that is a solution to each individual linear equation.

Example 1 Determining Solutions to a System of Linear Equations

Determine whether the ordered pairs are solutions to the system.

\[
\begin{align*}
  x + y &= -6 \\
  3x - y &= -2
\end{align*}
\]

**Solution:**

a. Substitute the ordered pair \((-2, -4)\) into both equations:

\[
\begin{align*}
  x + y &= -6 \\ 
  3x - y &= -2
\end{align*}
\]

\[
\begin{align*}
  (-2) + (-4) &= -6 & \text{True} \\
  3(-2) - (-4) &= -2 & \text{True}
\end{align*}
\]

Because the ordered pair \((-2, -4)\) is a solution to each equation, it is a solution to the system of equations.

b. Substitute the ordered pair \((0, -6)\) into both equations:

\[
\begin{align*}
  x + y &= -6 \\ 
  3x - y &= -2
\end{align*}
\]

\[
\begin{align*}
  (0) + (-6) &= -6 & \text{True} \\
  3(0) - (-6) &= 6 & \text{False}
\end{align*}
\]

Because the ordered pair \((0, -6)\) is not a solution to the second equation, it is not a solution to the system of equations.

Skill Practice Determine whether the ordered pairs are solutions to the system.

\[
\begin{align*}
  3x + 2y &= -8 \\
  y &= 2x - 18
\end{align*}
\]

1. \((-2, -1)\)  2. \((4, -10)\)

A solution to a system of two linear equations can be interpreted graphically as a point of intersection between the two lines.
Solving Systems of Linear Equations by the Graphing Method

Section 3.1  Solving Systems of Linear Equations by the Graphing Method

Graphing the lines from Example 1 we see that the point of intersection is \((-2, -4)\). Therefore, we say that the solution set is \(\{(-2, -4)\}\). See Figure 3-1.

2. Dependent and Inconsistent Systems of Linear Equations

When two lines are drawn in a rectangular coordinate system, three geometric relationships are possible:

1. Two lines may intersect at exactly one point.

2. Two lines may intersect at no point. This occurs if the lines are parallel.

3. Two lines may intersect at infinitely many points along the line. This occurs if the equations represent the same line (the lines are coinciding).

If a system of linear equations has one or more solutions, the system is said to be a consistent system. If a linear system has no solution, it is said to be an inconsistent system.

If two equations represent the same line, then all points along the line are solutions to the system of equations. In such a case, the system is characterized as a dependent system. An independent system is one in which the two equations represent different lines. The different possibilities for solutions to systems of linear equations are given in Table 3-1.

<table>
<thead>
<tr>
<th>Table 3-1 Solutions to Systems of Linear Equations in Two Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One Unique Solution</strong></td>
</tr>
<tr>
<td><img src="Image" alt="One point of intersection" /></td>
</tr>
<tr>
<td>System is consistent.</td>
</tr>
<tr>
<td>System is independent.</td>
</tr>
<tr>
<td><strong>No Solution</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Parallel lines" /></td>
</tr>
<tr>
<td>System is inconsistent.</td>
</tr>
<tr>
<td>System is independent.</td>
</tr>
<tr>
<td><strong>Infinitely Many Solutions</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Coinciding lines" /></td>
</tr>
<tr>
<td>System is consistent.</td>
</tr>
<tr>
<td>System is dependent.</td>
</tr>
</tbody>
</table>

3. Solving Systems of Linear Equations by Graphing

Example 2  Solving a System of Linear Equations by Graphing

Solve the system by graphing both linear equations and finding the point(s) of intersection.

\[
\begin{align*}
y &= \frac{1}{2}x - 2 \\
4x + 2y &= 6
\end{align*}
\]
Solution:

To graph each equation, write the equation in slope-intercept form $y = mx + b$.

**First equation**  
$y = \frac{1}{2}x - 2$  
Slope: $\frac{1}{2}$

**Second equation**  
$4x + 2y = 6$  
$2y = -4x + 6$  
$y = -2x + 3$  
Slope: $-2$

From their slope-intercept forms, we see that the lines have different slopes, indicating that the lines must intersect at exactly one point. We can graph the lines using the slope and $y$-intercept to find the point of intersection (Figure 3-2).

Avoiding Mistakes

Using graph paper may help you be more accurate when graphing lines. There are many websites from which you can print graph paper.

The point $(2, -1)$ appears to be the point of intersection. This can be confirmed by substituting $x = 2$ and $y = -1$ into both equations.

\[
\begin{align*}
y &= \frac{1}{2}x - 2 \\
4x + 2y &= 6 \\
4(2) + 2(-1) &= 6 \\
8 - 2 &= 6 \\
6 &= 6 \checkmark \text{ True}
\end{align*}
\]

The solution set is $\{(2, -1)\}$.

**Skill Practice** Solve by using the graphing method.

3. $y = -3x - 5$  
   $x - 2y = -4$

Answer

3. $\{(-2, 1)\}$
Section 3.1 Solving Systems of Linear Equations by the Graphing Method

Example 3  Solving a System of Linear Functions by Graphing

Solve the system.

\[ f(x) = 3 \]
\[ g(x) = 2x + 1 \]

Solution:

This first function can be written as \( y = 3 \). This is an equation of a horizontal line. Writing the second equation as \( y = 2x + 1 \), we have a slope of 2 and a \( y \)-intercept of \( (0, 1) \).

The graphs of the functions are shown in Figure 3-3. The point of intersection is \( (1, 3) \). Therefore, the solution set is \( \{(1, 3)\} \).

Skill Practice  Solve the system by graphing.

4. \( f(x) = 1 \)
   \[ g(x) = -3x + 4 \]

Example 4  Solving a System of Linear Equations by Graphing

Solve the system by graphing.

\[ -x + 3y = -6 \]
\[ 6y = 2x + 6 \]

Solution:

To graph the lines, write each equation in slope-intercept form.

\[ -x + 3y = -6 \]  \[ 6y = 2x + 6 \]
\[ 3y = x - 6 \]
\[ 6y = 2x + 6 \]
\[ \frac{3y}{3} = \frac{x}{3} - \frac{6}{3} \]
\[ \frac{6y}{6} = \frac{2x + 6}{6} \]
\[ y = \frac{1}{3}x - 2 \]
\[ y = \frac{1}{3}x + 1 \]

Because the lines have the same slope but different \( y \)-intercepts, they are parallel (Figure 3-4). Two parallel lines do not intersect, which implies that the system has no solution. The system is inconsistent.

The solution set is the empty set, \( \{ \} \).

Skill Practice  Solve the system by graphing.

5. \( 2y = 2x \)
   \[ -x + y = -3 \]

Answers

4. \( \{(1, 1)\} \)
5. The solution set is \( \{ \} \). The system is inconsistent.
Example 5  Solving a System of Linear Equations by Graphing

Solve the system by graphing.

\[ x + 4y = 8 \]
\[ y = -\frac{1}{4}x + 2 \]

Solution:
Write the first equation in slope-intercept form. The second equation is already in slope-intercept form.

First equation  
\[ 4y = -x + 8 \]
\[ y = -\frac{1}{4}x + 2 \]

Notice that the slope-intercept forms of the two lines are identical. Therefore, the equations represent the same line (Figure 3-5).

The system is dependent, and the solution to the system of equations is the set of all points on the line.

Because the ordered pairs in the solution set cannot all be listed, we can write the solution in set-builder notation. Furthermore, the equations \( x + 4y = 8 \) and \( y = -\frac{1}{4}x + 2 \) represent the same line. Therefore, the solution set may be written as \( \{(x, y) | y = -\frac{1}{4}x + 2\} \) or \( \{(x, y) | x + 4y = 8\} \).

Skill Practice  Solve the system by graphing.

6. \( y = \frac{1}{2}x + 1 \)
\( x - 2y = -2 \)

Calculator Connections

Topic: Using the Intersect Feature

The solution to a system of equations can be found by using either a Trace feature or an Intersect feature on a graphing calculator to find the point of intersection between two curves.

For example, consider the system
\[ -2x + y = 6 \]
\[ 5x + y = -1 \]
Section 3.1 Practice Exercises

Study Skills Exercises

1. Before you proceed further in Chapter 3, make your test corrections for the Chapter 2 test. See Exercise 1 of Section 2.1 for instructions.

2. Define the key terms.
   a. System of linear equations
   b. Solution to a system of linear equations
   c. Consistent system
   d. Inconsistent system
   e. Dependent system
   f. Independent system

Concept 1: Solutions to Systems of Linear Equations

For Exercises 3–8, determine which points are solutions to the given system. (See Example 1.)

3. \( y = 8x - 5 \)
   \( y = 4x + 3 \)
   \((-1, 13), (-1, 1), (2, 11)\)

4. \( y = -\frac{1}{2}x - 5 \)
   \( y = \frac{3}{4}x - 10 \)
   \((-4, -7), (0, -10), \left(3, -\frac{9}{2}\right)\)

5. \( 2x - 7y = -30 \)
   \( y = 3x + 7 \)
   \((0, -30), \left(\frac{3}{2}, 5\right), (-1, 4)\)
Chapter 3  Systems of Linear Equations and Inequalities

6. \( x + 2y = 4 \)
   \[ y = -\frac{1}{2}x + 2 \]
   \((-2, 3), (4, 0), \left(3, \frac{1}{2}\right)\)

7. \( x - y = 6 \)
   \[ 4x + 3y = -4 \]
   \((4, -2), (6, 0), (2, 4)\)

8. \( x - 3y = 3 \)
   \[ 2x - 9y = 1 \]
   \((0, 1), (4, -1), (9, 2)\)

Concept 2: Dependent and Inconsistent Systems of Linear Equations

For Exercises 9–14, the graph of a system of linear equations is given.

a. Identify whether the system is consistent or inconsistent.

b. Identify whether the system is dependent or independent.

c. Identify the number of solutions to the system.

9. \( y = x + 3 \)
   \[ 3x + y = -1 \]

10. \( 5x - 3y = 6 \)
    \[ 3y = 2x + 3 \]

11. \( 2x = y + 4 \)
    \[ -4x + 2y = 2 \]

12. \( y = -2x - 3 \)
    \[ -4x - 2y = 0 \]

13. \( y = \frac{1}{3}x + 2 \)
    \[ -x + 3y = 6 \]

14. \( y = -\frac{2}{3}x - 1 \)
    \[ -4x - 6y = 6 \]

Concept 3: Solving Systems of Linear Equations by Graphing

For Exercises 15–32, solve the systems of equations by graphing. (See Examples 2-5.)

15. \( 2x + y = -3 \)
    \[ -x + y = 3 \]

16. \( 4x - 3y = 12 \)
    \[ 3x + 4y = -16 \]

17. \( f(x) = -2x + 3 \)
    \[ g(x) = 5x - 4 \]
Section 3.1  Solving Systems of Linear Equations by the Graphing Method

18. \( h(x) = 2x + 5 \)
    \( g(x) = -x + 2 \)

19. \( k(x) = \frac{1}{3}x - 5 \)
    \( f(x) = -\frac{2}{3}x - 2 \)

20. \( f(x) = \frac{1}{2}x + 2 \)
    \( g(x) = \frac{5}{2}x - 2 \)

21. \( x = 4 \)
    \( y = 2x - 3 \)

22. \( 3x + 2y = 6 \)
    \( y = -3 \)

23. \( y = -2x + 3 \)
    \(-2x = y + 1 \)

24. \( y = \frac{1}{3}x - 2 \)
    \( x = 3y - 9 \)

25. \( y = \frac{2}{3}x - 1 \)
    \( 2x = 3y + 3 \)

26. \( 4x = 16 - 8y \)
    \( y = -\frac{1}{2}x + 2 \)
Chapter 3  Systems of Linear Equations and Inequalities

27.  \[2x = 4\]  \[\frac{1}{2}y = -1\]

28.  \[y + 7 = 6\]  \[-5 = 2x\]

29.  \[-x + 3y = 6\]  \[6y = 2x + 12\]

30.  \[3x = 2y - 4\]  \[-4y = -6x - 8\]

31.  \[2x - y = 4\]  \[4x + 2 = 2y\]

32.  \[x = 4y + 4\]  \[-2x + 8y = -16\]

For Exercises 33–36, identify each statement as true or false.

33. A consistent system is a system that always has a unique solution.

34. A dependent system is a system that has no solution.

35. If two lines coincide, the system is dependent.

36. If two lines are parallel, the system is independent.

Graphing Calculator Exercises

For Exercises 37–42, use a graphing calculator to graph each linear equation on the same viewing window. Use a Trace or Intersect feature to find the point(s) of intersection.

37.  \[y = 2x - 3\]  \[y = -4x + 9\]

38.  \[y = -\frac{1}{2}x + 2\]  \[y = \frac{1}{3}x - 3\]

39.  \[x + y = 4\]  \[-2x + y = -5\]

40.  \[x - 2y = -2\]  \[-3x + 2y = 6\]

41.  \[-x + 3y = -6\]  \[6y = 2x + 6\]

42.  \[x + 4y = 8\]  \[y = \frac{1}{4}x + 2\]
Solving Systems of Linear Equations by the Substitution Method

1. The Substitution Method

Graphing a system of equations is one method to find the solution of the system. However, sometimes it is difficult to determine the solution using this method because of limitations in the accuracy of the graph. This is particularly true when the coordinates of a solution are not integer values or when the solution is a point not sufficiently close to the origin. Identifying the coordinates of the point \((\frac{1}{2}, \frac{1}{2})\) or \((-251, 8349)\), for example, might be difficult from a graph.

In this section and Section 3.3, we will present two algebraic methods to solve a system of equations. The first is called the substitution method. This technique is particularly important because it can be used to solve more advanced problems including nonlinear systems of equations.

The first step in the substitution process is to isolate one of the variables from one of the equations. Consider the system

\[
\begin{align*}
x + y &= 16 \\
x - y &= 4
\end{align*}
\]

Solving the first equation for \(x\) yields \(x = 16 - y\). Then, because \(x\) is equal to \(16 - y\), the expression \(16 - y\) may replace \(x\) in the second equation. This leaves the second equation in terms of \(y\) only.

First equation: \(x + y = 16\) \(\rightarrow\) \(x = 16 - y\)

Second equation: \((16 - y) - y = 4\) \(\rightarrow\) \(16 - 2y = 4\) \(\rightarrow\) \(-2y = -12\) \(\rightarrow\) \(y = 6\)

\(x = 16 - y\) \(\rightarrow\) \(x = 16 - 6\) \(\rightarrow\) \(x = 10\)

To find \(x\), substitute \(y = 6\) back into the expression \(x = 16 - y\).

\(x = 16 - (6)\) \(\rightarrow\) \(x = 10\)

Check the ordered pair \((10, 6)\) in both original equations.

\(x + y = 16\) \(\rightarrow\) \((10) + (6) = 16\) \(\checkmark\) True

\(x - y = 4\) \(\rightarrow\) \((10) - (6) = 4\) \(\checkmark\) True

The solution set is \{\((10, 6)\)\}. 
Chapter 3 Systems of Linear Equations and Inequalities

**PROCEDURE**  
**Solving a System of Equations by the Substitution Method**

- **Step 1**  
  Isolate one of the variables from one equation.

- **Step 2**  
  Substitute the quantity found in step 1 into the other equation.

- **Step 3**  
  Solve the resulting equation.

- **Step 4**  
  Substitute the value found in step 3 back into the equation in step 1 to find the value of the remaining variable.

- **Step 5**  
  Check the solution in both equations, and write the answer as an ordered pair within set notation.

**Example 1**  
**Using the Substitution Method to Solve a System of Linear Equations**

Solve the system by using the substitution method.

\[3x - 2y = -7\]

\[6x + y = 6\]

**Solution:**

The \(y\) variable in the second equation is the easiest variable to isolate because its coefficient is 1.

\[3x - 2y = -7\]

\[6x + y = 6\]

**Step 1:** Solve the second equation for \(y\).

\[y = -6x + 6\]

**Step 2:** Substitute the quantity \(-6x + 6\) for \(y\) in the other equation.

\[3x - 2(-6x + 6) = -7\]

\[3x + 12x - 12 = -7\]

\[15x - 12 = -7\]

\[15x = 5\]

\[x = \frac{1}{3}\]

**Step 3:** Solve for \(x\).

\[y = -6x + 6\]

\[y = -6\left(\frac{1}{3}\right) + 6\]

\[y = -2 + 6\]

\[y = 4\]

**Step 4:** Substitute \(x = \frac{1}{3}\) into the expression \(y = -6x + 6\).

\[3\left(\frac{1}{3}\right) - 2(4) = -7\]

\[6\left(\frac{1}{3}\right) + 4 = 6\]

\[1 - 8 \neq -7\]

\[2 + 4 = 6\]

The solution set is \(\{(\frac{1}{3}, 4)\}\).
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

**Skill Practice** Solve by using the substitution method.

1. \(3x + y = 8\)
   \(x - 2y = 12\)

**Example 2** Using the Substitution Method to Solve a System of Linear Equations

Solve the system by using the substitution method.

\[-3x + 4y = 9\]
\[x = \frac{1}{3}y + 2\]

**Solution:**

\[-3x + 4y = 9\]
\[x = \frac{1}{3}y + 2\]

**Step 1:** In the second equation, \(x\) is already isolated.

\[-3\left(\frac{1}{3}y + 2\right) + 4y = 9\]

**Step 2:** Substitute the quantity \(\frac{1}{3}y + 2\) for \(x\) in the other equation.

\[y - 6 + 4y = 9\]
\[5y = 15\]
\[y = 3\]

Now use the known value of \(y\) to solve for the remaining variable \(x\).

\[x = \frac{1}{3}y + 2\]
\[x = \frac{1}{3}(3) + 2\]
\[x = -1 + 2\]
\[x = 1\]

**Step 5:** Check the ordered pair \((1, 3)\) in each original equation.

\[-3x + 4y = 9\]
\[-3(1) + 4(3) = 9\]
\[-3 + 12 = 9 \checkmark True\]

\[x = \frac{1}{3}y + 2\]
\[1 = \frac{1}{3}(3) + 2\]
\[1 = -1 + 2 \checkmark True\]

The solution set is \{(1, 3)\}.

**Skill Practice** Solve by using the substitution method.

2. \(x = 2y + 3\)
   \(4x - 2y = 0\)

**Answers**

1. \((4, -4)\)  
2. \((-1, -2)\)
2. Solving Inconsistent Systems and Dependent Systems

**Example 3** Solving an Inconsistent System

Solve the system by using the substitution method.

\[ x = 2y - 4 \]
\[ -2x + 4y = 6 \]

**Solution:**

\[ x = 2y - 4 \]
\[ -2x + 4y = 6 \]

**Step 1:** The \( x \) variable is already isolated.

**Step 2:** Substitute the quantity \( x = 2y - 4 \) into the other equation.

\[ -2(2y - 4) + 4y = 6 \]
\[ -4y + 8 + 4y = 6 \]
\[ 8 = 6 \]

There is no solution.
The system is inconsistent.
The solution set is \( \{ \} \).

**TIP:** The answer to Example 3 can be verified by writing each equation in slope-intercept form and graphing the equations.

**Skill Practice** Solve by using the substitution method.

3. \( 8x - 16y = 3 \)
\[ y = \frac{1}{2}x + 1 \]

**Answer**

3. No solution; \( \{ \} \); inconsistent system
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

Example 4 Solving a Dependent System

Solve by using the substitution method.

\[ \begin{align*}
4x - 2y &= -6 \\
y - 3 &= 2x
\end{align*} \]

Solution:

\[ \begin{align*}
4x - 2y &= -6 \\
y - 3 &= 2x & \text{Step 1: Solve for one of the variables.} \\
4x - 2(2x + 3) &= -6 & \text{Step 2: Substitute the quantity } 2x + 3 \text{ for } y \text{ in the other equation.} \\
4x - 4x - 6 &= -6 & \text{Step 3: Solve for } x. \text{ Apply the distributive property to clear the parentheses.}
\end{align*} \]

The system reduces to the identity \(-6 = -6\). Therefore, the original two equations are equivalent, and the system is dependent. The solution consists of all points on the common line, giving us an infinite number of solutions. Because the equations \(4x - 2y = -6\) and \(y - 3 = 2x\) represent the same line, the solution set is

\[ \{(x, y) | 4x - 2y = -6\} \quad \text{or} \quad \{(x, y) | y - 3 = 2x\} \]

Skill Practice Solve the system by using substitution.

4. \(3x + 6y = 12\) \quad 2y = -x + 4

TIP: We can confirm the results of Example 4 by writing each equation in slope-intercept form. The slope-intercept forms are identical, indicating that the lines are the same.

\[ \begin{align*}
4x - 2y &= -6 & \Rightarrow & & y &= 2x + 3 \\
y - 3 &= 2x & \Rightarrow & & y &= 2x + 3
\end{align*} \]

Answer

4. Infinitely many solutions; \(\{(x, y) | 3x + 6y = 12\}\); dependent system

Section 3.2 Practice Exercises

For Exercises 1–4, use the slope-intercept form of the lines to determine the number of solutions to the system.

1. \(y = 8x - 1\) \quad \(2x - 16y = 3\)
2. \(4x + 6y = 1\) \quad \(10x + 15y = \frac{5}{2}\)
3. \(2x - 4y = 0\) \quad \(x - 2y = 9\)
4. \(6x + 3y = 8\) \quad \(8x + 4y = -1\)
Chapter 3  Systems of Linear Equations and Inequalities

5. Determine if the ordered pair \((-4, 3)\) is a solution to the system.
   \[-x + 2y = 10\]
   \[2x - y = 11\]

For Exercises 6–7, solve the system by graphing.

6. \[
x - y = 4 \\
3x + 4y = 12
\]

7. \[
y = 2x + 3 \\
6x + 3y = 9
\]

Concept 1: The Substitution Method

For Exercises 8–21, solve by using the substitution method. (See Examples 1–2.)

8. \[4x + 12y = 4\]
   \[y = 5x + 11\]

9. \[y = -3x - 1\]
   \[2x - 3y = -8\]

10. \[10y + 34 = x\]
   \[-7x + y = -31\]

11. \[-3x + 8y = -1\]
   \[4x - 11 = y\]

12. \[12x - 2y = 0\]
   \[-7x + 2y = -1\]

13. \[3x + 12y = 36\]
   \[x + 5y = 12\]

14. \[x - 3y = -3\]
   \[2x + 3y = -6\]

15. \[x - y = 8\]
   \[3x + 2y = 9\]

16. \[5x - 2y = 10\]
   \[y = x - 1\]

17. \[2x - y = -1\]
   \[y = -2x\]

18. \[1 + 3y = 10\]
   \[5x + 2y = 6\]

19. \[2x + 3 = 7\]
   \[3x - 4y = 6\]

20. \[2x + 3y = 7\]
   \[-5x = 2y - 12\]

21. \[4x - 5y = 14\]
   \[3y = x - 7\]

22. Describe the process of solving a system of linear equations by using substitution.

Concept 2: Solving Inconsistent Systems and Dependent Systems

For Exercises 23–30, solve the systems. (See Examples 3–4.)

23. \[2x - 6y = -2\]
   \[x = 3y - 1\]

24. \[-2x + 4y = 22\]
   \[x = 2y - 11\]

25. \[y = \frac{1}{7}x + 3\]
   \[x - 7y = -4\]

26. \[x = \frac{-3}{2}y + \frac{1}{2}\]
   \[4x + 6y = 7\]

27. \[5x - y = 10\]
   \[2y = 10x - 5\]

28. \[x + 4y = 8\]
   \[3x = 3 - 12y\]

29. \[3x - y = 7\]
   \[-14 + 6x = 2y\]

30. \[x = 4y + 1\]
   \[-12y = -3x + 3\]

31. When using the substitution method, explain how to determine whether a system of linear equations is dependent.

32. When using the substitution method, explain how to determine whether a system of linear equations is inconsistent.
Mixed Exercises
For Exercises 33–58, solve the system by using the substitution method.

33. \( x = 1.3y + 1.5 \\
    y = 1.2x - 4.6 \\
34. \ y = 0.8x - 1.8 \\
    1.1x = -y + 9.6 \\
35. \ y = \frac{2}{3}x - \frac{1}{3} \\
    x = \frac{1}{4}y + \frac{17}{4} \\
36. \ x = \frac{1}{5}y - \frac{5}{3} \\
    y = \frac{1}{5}x + \frac{21}{5} \\
37. \ -2x + y = 4 \\
    -\frac{1}{4}x + \frac{1}{8}y = \frac{1}{4} \\
38. \ 8x - y = 8 \\
39. \ 3x + 2y = 6 \\
    y = x + 3 \\
40. \ -x + 4y = -4 \\
    y = x - 1 \\
41. \ -300x - 125y = 1350 \\
    y + 2 = 8 \\
42. \ 200y = 150x \\
    y - 4 = 1 \\
43. \ 2x - y = 6 \\
    \frac{1}{6}x - \frac{1}{12}y = \frac{1}{2} \\
44. \ x - 4y = 8 \\
45. \ y = 200x - 320 \\
    y = -150x + 1080 \\
46. \ y = -54x + 300 \\
    y = 20x - 70 \\
47. \ y = -2.7x - 5.1 \\
    y = 3.1x + 63.1 \\
48. \ y = 6.8x + 2.3 \\
    y = -4.1x + 56.8 \\
49. \ 4x + 4y = 5 \\
    x - 4y = -\frac{5}{2} \\
50. \ -2x + y = -6 \\
    6x - 13y = -12 \\
51. \ 2(x + 2y) = 12 \\
    -6x = 5y - 8 \\
52. \ 5x - 2y = -25 \\
    10x = 3(y - 10) \\
53. \ 5(3y - 2) = x + 4 \\
    4y = 7x - 3 \\
54. \ 2x = -3(y + 3) \\
    3x - 4y = -22 \\
55. \ 2x - 5 = 7 \\
    4 = 3y + 1 \\
56. \ -2 = 4 - 2y \\
    7x - 5 = -5 \\
57. \ 0.01y = 0.02x - 0.11 \\
    0.3x - 0.5y = 2 \\
58. \ 0.3x - 0.4y = 1.3 \\
    0.01x = 0.03y + 0.01
Avoiding Mistakes
Be sure to multiply both sides of the equation by 4:

\[ 4(4x + 9) = 4(9) \]

Example 1  Solving a System by the Addition Method

Solve the system by using the addition method.

\[ \begin{align*}
3x - 4y &= 2 \\
4x + y &= 9
\end{align*} \]

**Solution:**

\[ \begin{align*}
3x - 4y &= 2 \\
4x + y &= 9
\end{align*} \]

Multiply the second equation by 4. This makes the coefficients of the \( y \) variables opposite.

Now if the equations are added, the \( y \) variable will be eliminated.

\[ \begin{align*}
3x - 4y &= 2 \\
16x + 4y &= 36
\end{align*} \]

\[ 19x = 38 \]

\[ x = 2 \]

Solve for \( x \).

\[ 3(2) - 4y = 2 \]

Substitute \( x = 2 \) back into one of the original equations and solve for \( y \).

\[ 6 - 4y = 2 \]

\[ -4y = -4 \]

\[ y = 1 \]

Check the ordered pair \((2, 1)\) in each original equation:

\[ \begin{align*}
3(2) - 4(1) &\neq 2 \checkmark \text{ True} \\
4(2) + (1) &\neq 9 \checkmark \text{ True}
\end{align*} \]

The solution set is \( \{(2, 1)\} \).

Skill Practice  Solve by using the addition method.

1. \( 2x - 3y = 13 \)
   \[ x + 2y = 3 \]

The steps to solve a system of linear equations in two variables by the addition method is outlined in the following box.

**PROCEDURE  Solving a System of Linear Equations by the Addition Method**

**Step 1**  Write both equations in standard form: \( Ax + By = C \).

**Step 2**  Clear fractions or decimals (optional).

**Step 3**  Multiply one or both equations by nonzero constants to create opposite coefficients for one of the variables.

**Step 4**  Add the equations from step 3 to eliminate one variable.

**Step 5**  Solve for the remaining variable.

**Step 6**  Substitute the known value found in step 5 into one of the original equations to solve for the other variable.

**Step 7**  Check the ordered pair in both equations and write the solution set.
Example 2 Solving a System by the Addition Method

Solve the system by using the addition method.

\[
\begin{align*}
4x + 5y &= 2 \\
3x &= 1 - 4y
\end{align*}
\]

Solution:

\[
\begin{align*}
4x + 5y &= 2 & \text{Step 1: Write both equations in} \\
3x + 4y &= 1 & \text{standard form. There are no} \\
& & \text{fractions or decimals.}
\end{align*}
\]

We may choose to eliminate either variable. To eliminate \(x\), change the coefficients to 12 and \(-12\).

\[
\begin{align*}
4x + 5y &= 2 & \text{Multiply by 3:} \\
3x + 4y &= 1 & \text{Multiply by } -4
\end{align*}
\]

\[
\begin{align*}
12x + 15y &= 6 & \text{Step 3: Multiply the first} \\
-12x - 16y &= -4 & \text{equation by 3.} \\
\hline
-12x - 16y &= -4 & \text{Multiply the second} \\
\end{align*}
\]

\[
\begin{align*}
-31y &= 2 & \text{Step 4: Add the equations.} \\
y &= -2 & \text{Step 5: Solve for } y.
\end{align*}
\]

\[
\begin{align*}
4x + 5y &= 2 \\
4x + 5(-2) &= 2 \\
4x - 10 &= 2 \\
4x &= 12 \\
x &= 3
\end{align*}
\]

The solution set is \((3, -2)\).  

Step 6: Substitute \(y = -2\) back into one of the original equations and solve for \(x\).

Step 7: Check the ordered pair \((3, -2)\) in both original equations.

TIP: To eliminate the \(x\) variable in Example 2, both equations were multiplied by appropriate constants to create \(12x\) and \(-12x\). We chose 12 because it is the least common multiple of 4 and 3.

We could have solved the system by eliminating the \(y\) variable. To eliminate \(y\), we would multiply the top equation by 4 and the bottom equation by \(-5\). This would make the coefficients of the \(y\) variable 20 and \(-20\), respectively.

\[
\begin{align*}
4x + 5y &= 2 & \text{Multiply by 4:} \\
3x + 4y &= 1 & \text{Multiply by } -5
\end{align*}
\]

\[
\begin{align*}
16x + 20y &= 8 & \text{16x - 20y = 8} \\
-15x - 20y &= -5 & \text{16x - 20y = 8}
\end{align*}
\]

Skill Practice Solve by using the addition method.

2. \(2y = 5x - 4\)  
\(3x - 4y = 1\)

Answer

2. \(\left\{ \left(1, \frac{1}{2} \right) \right\} \)
Chapter 3 Systems of Linear Equations and Inequalities

Example 3 Solving a System by the Addition Method

Solve the system by using the addition method.

\[
\begin{align*}
x - 2y &= 6 + y \\
0.05y &= 0.02x - 0.10
\end{align*}
\]

Solution:

\[
\begin{align*}
x - 2y &= 6 + y \\
0.05y &= 0.02x - 0.10
\end{align*} \quad \rightarrow \quad \begin{align*}
x - 3y &= 6 \\
-0.02x + 0.05y &= -0.10
\end{align*}
\]

Multiply by 100:

\[
\begin{align*}
x - 3y &= 6 \\
-2x + 5y &= -10
\end{align*}
\]

Step 1: Write both equations in standard form.

Step 2: Clear decimals.

Step 3: Create opposite coefficients.

Step 4: Add the equations.

Step 5: Solve for y.

\[
\begin{align*}
x - 2y &= 6 + y \\
x - 2(-2) &= 6 + (-2) \\
x + 4 &= 4 \\
x &= 0
\end{align*}
\]

Step 6: To solve for x, substitute \(y = -2\) into one of the original equations.

Step 7: The ordered pair \((0, -2)\) checks in each original equation.

The solution set is \(\{(0, -2)\}\).

Skill Practice Solve by using the addition method.

3. \(0.2x + 0.3y = 1.5\)
   \(5x + 3y = 20 - y\)

2. Solving Inconsistent Systems and Dependent Systems

Example 4 Solving a Dependent System

Solve the system by using the addition method.

\[
\begin{align*}
\frac{1}{5}x - \frac{1}{2}y &= 1 \\
-4x + 10y &= -20
\end{align*}
\]

Solution:

\[
\begin{align*}
\frac{1}{5}x - \frac{1}{2}y &= 1 \\
-4x + 10y &= -20
\end{align*} \quad \text{Step 1: Equations are in standard form.}
\]

Answer

3. \(\{(0, 5)\}\)
Notice that both variables were eliminated. The system of equations is reduced to the identity \(0 = 0\). Therefore, the two original equations are equivalent and the system is dependent. The solution set consists of an infinite number of ordered pairs \((x, y)\) that fall on the common line of intersection \(-4x + 10y = -20\), or equivalently \(\frac{1}{2}x - \frac{1}{2}y = 1\). The solution set is

\[
\{(x, y) \mid -4x + 10y = -20\} \quad \text{or} \quad \left\{(x, y) \mid \frac{1}{2}x - \frac{1}{2}y = 1\right\}
\]

**Skill Practice** Solve by the addition method.

4. \(3x + y = 4\)
   \(x + \frac{1}{3}y = \frac{4}{3}\)

**Example 5** Solving an Inconsistent System

Solve the system by using the addition method.

\[
2y = -3x + 4
\]

Step 1: Write the equations in standard form.

\[
2y = -3x + 4 \quad \rightarrow \quad 3x + 2y = 4
\]

\[
20(6x + 5y) = 40 + 20y
\]

Step 2: There are no decimals or fractions.

\[
3x + 2y = 4 \quad \rightarrow \quad -120x - 80y = -160
\]

\[
120x + 80y = 40
\]

The equations reduce to a contradiction, indicating that the system has no solution. The system is inconsistent. The two equations represent parallel lines, as shown in Figure 3-8.

There is no solution, \(\emptyset\).

**Skill Practice** Solve by using the addition method.

5. \(18 + 10x = 6y\)
   \(5x - 3y = 9\)
Chapter 3  Systems of Linear Equations and Inequalities

Section 3.3  Practice Exercises

Review Exercises
For Exercises 1–4, use the slope-intercept form of the lines to determine the number of solutions for the system of equations.

1. \[ y = \frac{1}{2}x - 4 \]
2. \[ y = 2.32x - 8.1 \]
3. \[ 4x = y + 7 \]
4. \[ 3x - 2y = 9 \]

Concept 1: The Addition Method
For Exercises 5–16, solve the system by using the addition method. (See Examples 1–3.)

5. \[ \begin{align*} 3x - y &= -1 \\ -3x + 4y &= -14 \end{align*} \]
6. \[ \begin{align*} 5x - 2y &= 15 \\ 3x + 2y &= -7 \end{align*} \]
7. \[ \begin{align*} 2x + 3y &= 3 \\ 10x + 2y &= -32 \end{align*} \]
8. \[ \begin{align*} 2x - 5y &= 7 \\ 3x - 10y &= 13 \end{align*} \]
9. \[ \begin{align*} 3x + 7y &= -20 \\ -5x + 3y &= -84 \end{align*} \]
10. \[ \begin{align*} 6x - 9y &= -15 \\ 5x - 2y &= -40 \end{align*} \]
11. \[ \begin{align*} 3x &= 10y + 13 \\ 7y &= 4x - 11 \end{align*} \]
12. \[ \begin{align*} -5x &= 6y - 4 \\ 5y &= 1 - 3x \end{align*} \]
13. \[ \begin{align*} 1.2x - 0.6y &= 3 \\ 0.8x - 1.4y &= 3 \end{align*} \]
14. \[ \begin{align*} 1.8x + 0.8y &= 1.4 \\ 1.2x + 0.6y &= 1.2 \end{align*} \]
15. \[ \begin{align*} 3x + 2 &= 4y + 2 \\ 7x &= 3y \end{align*} \]
16. \[ \begin{align*} -4y - 3 &= 2x - 3 \\ 5y &= 3x \end{align*} \]

Concept 2: Solving Inconsistent Systems and Dependent Systems
For Exercises 17–24, solve the systems. (See Examples 4–5.)

17. \[ \begin{align*} 3x - 2y &= 1 \\ -6x + 4y &= -2 \end{align*} \]
18. \[ \begin{align*} 3x - y &= 4 \\ 6x - 2y &= 8 \end{align*} \]
19. \[ \begin{align*} 6y &= 14 - 4x \\ 2x &= -3y - 7 \end{align*} \]
20. \[ \begin{align*} 2x &= 4 - y \\ -y &= 2x - 2 \end{align*} \]
21. \[ \begin{align*} 12x - 4y &= 2 \\ 6x &= 1 + 2y \end{align*} \]
22. \[ \begin{align*} 10x - 15y &= 5 \\ 3y &= 2x - 1 \end{align*} \]
23. \[ \begin{align*} \frac{1}{2}x + y &= \frac{7}{6} \\ x + 2y &= 4.5 \end{align*} \]
24. \[ \begin{align*} 0.2x - 0.1y &= -1.2 \\ x - \frac{1}{2}y &= 3 \end{align*} \]

Mixed Exercises
25. Describe a situation in which you would prefer to use the substitution method over the addition method.

26. If you used the addition method to solve the given system, would it be easier to eliminate the \( x \) or \( y \) variable? Explain.

\[ \begin{align*} 3x - 5y &= 4 \\ 7x + 10y &= 31 \end{align*} \]
Section 3.3  Solving Systems of Linear Equations by the Addition Method

For Exercises 27–52, solve by using either the addition method or the substitution method.

27. $2x - 4y = 8$
   \[ y = 2x + 1 \]

28. $8x + 6y = -8$
   \[ x = 6y - 10 \]

29. $2x + 5y = 9$
   \[ 4x - 7y = -16 \]

30. \[ 0.1x + 0.5y = 0.7 \]
   \[ 0.2x + 0.7y = 0.8 \]

31. \[ 0.2x - 0.1y = 0.8 \]
   \[ 0.1x - 0.1y = 0.4 \]

32. \[ y = \frac{1}{2}x - 3 \]
   \[ 4x + y = -3 \]

33. $4x - 6y = 5$
   \[ 2x - 3y = 7 \]

34. $3x + 6y = 7$
   \[ 2x + 4y = 5 \]

35. \[ \frac{1}{4}x - \frac{1}{6}y = -2 \]
   \[ -\frac{1}{6}x + \frac{1}{5}y = 4 \]

36. \[ \frac{1}{3}x + \frac{1}{5}y = 7 \]
   \[ \frac{1}{6}x - \frac{2}{5}y = -4 \]

37. \[ \frac{1}{3}x - \frac{1}{2}y = 0 \]
   \[ x = \frac{3}{2}y \]

38. \[ \frac{2}{5}x - \frac{2}{3}y = 0 \]
   \[ y = \frac{3}{5}y \]

39. $2(x + 2y) = 20 - y$
   \[ -7(x - y) = 16 + 3y \]

40. \[ -3(x + y) = 10 - 4y \]
   \[ 4(x + 2y) = 50 + 3y \]

41. \[ -4y = 10 \]
   \[ 4x + 3 = 1 \]

42. \[ -9x = 15 \]
   \[ 3y + 2 = 1 \]

43. \[ 0.04x = -0.05y + 1.7 \]
   \[ -0.03y = -2.4 + 0.07x \]

44. \[ -0.01x = -0.06y + 3.2 \]
   \[ 0.08y = 0.03x + 4.6 \]

45. \[ 3x - 2 = \frac{1}{3}(11 + 5y) \]
   \[ x + \frac{2}{3}(2y - 3) = -2 \]

46. \[ 2(2y + 3) - 2x = 1 - x \]
   \[ x + y = \frac{1}{5}(7 + y) \]

47. \[ \frac{1}{4}x + \frac{1}{2}y = \frac{11}{4} \]
   \[ \frac{2}{3}x + \frac{1}{3}y = \frac{7}{3} \]

48. \[ \frac{1}{10}x - \frac{1}{2}y = -\frac{8}{5} \]
   \[ x + \frac{1}{4}y = -\frac{11}{2} \]

49. \[ 4x + y = -2 \]
   \[ 5x - y = -7 \]

50. \[ 4y = 8x + 20 \]
   \[ 8x = 24 \]

51. \[ 4x = 3y \]
   \[ y = \frac{4}{3}x + 2 \]

52. \[ 4x - 2y = 6 \]
   \[ x = \frac{1}{2}y + \frac{3}{2} \]

Expanding Your Skills

Sometimes the solution to a system of equations is an ordered pair containing fractions. In such a case, it is often easier to solve for each coordinate separately using the addition method. That is, use the addition method to solve for $x$. Then, rather than substituting $x$ back into one of the original equations, repeat the addition method and solve for $y$. For Exercises 53–55, solve each system.

53. \[ 9x + 11y = 47 \]
   \[ -5x + 3y = 23 \]

54. \[ -6x + 7y = -4 \]
   \[ 4x - 9y = 31 \]

55. \[ 4x - 10y = 19 \]
   \[ 5x + 12y = -41 \]
Chapter 3  Systems of Linear Equations and Inequalities

Problem Recognition Exercises

Solving Systems of Linear Equations
For Exercises 1–4, solve each system by using three different methods.

a. Use the graphing method.
b. Use the substitution method.
c. Use the addition method.

1. \(-3x + y = -2\) \hspace{1cm} 2. \(3x - 2y = 4\) \hspace{1cm} 3. \(5x = 2y\) \hspace{1cm} 4. \(2y = 3x + 1\)
   
   \(4x - y = 4\) \hspace{1cm} \(x = \frac{2}{3}y - 2\) \hspace{1cm} \(y = \frac{5}{2}x + 1\) \hspace{1cm} \(4x = 4\)

For Exercises 5–8, use the systems of equations shown. Use each choice only once.

a. \(y = -4x - 9\) \hspace{1cm} b. \(5x - 2y = -17\) \hspace{1cm} c. \(5x - 3y = 2\) \hspace{1cm} d. \(\frac{1}{10}x - \frac{2}{5}y = -\frac{3}{5}\)
   
   \(8x + 3y = -29\) \hspace{1cm} \(x + 5y = 2\) \hspace{1cm} \(7x + 4y = -30\) \hspace{1cm} \(\frac{3}{4}x + \frac{1}{3}y = \frac{13}{6}\)

5. For which system would you clear fractions? Solve the system.
6. Which system would be solved most efficiently by using the substitution method? Solve the system.
7. Which system would be solved most efficiently by using the addition method? Solve the system.
8. Which system would be solved efficiently using either the addition or the substitution method? Solve the system.

Section 3.4  Applications of Systems of Linear Equations in Two Variables

Concepts

1. Applications Involving Cost
2. Applications Involving Mixtures
3. Applications Involving Principal and Interest
4. Applications Involving Uniform Motion
5. Applications Involving Geometry

1. Applications Involving Cost

In Chapter 1 we solved numerous application problems using equations that contained one variable. However, when an application has more than one unknown, sometimes it is more convenient to use multiple variables. In this section, we will solve applications containing two unknowns. When two variables are present, the goal is to set up a system of two independent equations.
Section 3.4 Applications of Systems of Linear Equations in Two Variables

Example 1 Solving a Cost Application

At an amusement park, five hot dogs and one drink cost $16. Two hot dogs and three drinks cost $9. Find the cost per hot dog and the cost per drink.

Solution:

Let \( h \) represent the cost per hot dog. Label the variables.

Let \( d \) represent the cost per drink.

\[
\begin{align*}
\text{Cost of 5 hot dogs} + (\text{cost of 1 drink}) &= 16 \\
5h + d &= 16
\end{align*}
\]
Write two equations.

\[
\begin{align*}
\text{Cost of 2 hot dogs} + (\text{cost of 3 drinks}) &= 9 \\
2h + 3d &= 9
\end{align*}
\]

This system can be solved by either the substitution method or the addition method. We will solve by using the substitution method. The \( d \) variable in the first equation is the easiest variable to isolate.

\[
\begin{align*}
5h + d &= 16 \\
d &= -5h + 16
\end{align*}
\]
Solve for \( d \) in the first equation.

\[
\begin{align*}
2h + 3d &= 9 \\
2h + 3(-5h + 16) &= 9 \\
2h - 15h + 48 &= 9 \\
-13h + 48 &= 9 \\
-13h &= -39 \\
h &= 3
\end{align*}
\]
Clear parentheses.

\[
\begin{align*}
d &= -5(3) + 16 \\
d &= 1
\end{align*}
\]
Substitute \( h = 3 \) in the equation \( d = -5h + 16 \).

Because \( h = 3 \), the cost per hot dog is $3.00.

Because \( d = 1 \), the cost per drink is $1.00.

Skill Practice

1. At the movie theater, Tom spent $15.50 on 3 soft drinks and 2 boxes of popcorn. Carly bought 5 soft drinks and 1 box of popcorn for a total of $16.50. Use a system of equations to find the cost of a soft drink and the cost of a box of popcorn.

TIP: A word problem can be checked by verifying that the solution meets the conditions specified in the problem.

\[
\begin{align*}
5 \text{ hot dogs} + 1 \text{ drink} &= 5(3.00) + 1(1.00) = 16.00 \checkmark \\
2 \text{ hot dogs} + 3 \text{ drinks} &= 2(3.00) + 3(1.00) = 9.00 \checkmark
\end{align*}
\]

Answer

1. Soft drinks cost $2.50 and popcorn costs $4.00.
2. Applications Involving Mixtures

Example 2  Solving an Application Involving Chemistry

One brand of cleaner used to etch concrete is 25% acid. A stronger industrial-strength cleaner is 50% acid. How many gallons of each cleaner should be mixed to produce 20 gal of a 40% acid solution?

Solution:

Let $x$ represent the amount of 25% acid cleaner.

Let $y$ represent the amount of 50% acid cleaner.

<table>
<thead>
<tr>
<th></th>
<th>25% Acid</th>
<th>50% Acid</th>
<th>40% Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of gallons of solution</td>
<td>$x$</td>
<td>$y$</td>
<td>20</td>
</tr>
<tr>
<td>Number of gallons of pure acid</td>
<td>0.25$x$</td>
<td>0.50$y$</td>
<td>0.40(20), or 8</td>
</tr>
</tbody>
</table>

From the first row of the table, we have

\[
\left( \frac{\text{Amount of}}{25\% \text{ solution}} \right) + \left( \frac{\text{amount of}}{50\% \text{ solution}} \right) = \left( \frac{\text{total amount}}{\text{of solution}} \right) \implies x + y = 20
\]

From the second row of the table we have

\[
\left( \frac{\text{Amount of}}{\text{pure acid in}} \right) + \left( \frac{\text{amount of}}{\text{pure acid in}} \right) = \left( \frac{\text{amount of}}{\text{resulting solution}} \right) \implies 0.25x + 0.50y = 8
\]

Multiply by 100 to clear decimals.

\[
x + y = 20 \quad \implies -25x - 25y = -500
\]

Create opposite coefficients of $x$.

\[
25x + 50y = 800 \quad \implies 25y = 300
\]

Add the equations to eliminate $x$.

\[
y = 12
\]

Substitute $y = 12$ back into one of the original equations.

\[
x + (12) = 20 \implies x = 8
\]

Therefore, 8 gal of 25% acid solution must be added to 12 gal of 50% acid solution to create 20 gal of a 40% acid solution.

Skill Practice

2. A pharmacist needs 8 ounces (oz) of a solution that is 50% saline. How many ounces of 60% saline solution and 20% saline solution must be mixed to obtain the mixture needed?

Answer

2. The pharmacist should mix 6 oz of 60% solution and 2 oz of 20% solution.
### 3. Applications Involving Principal and Interest

#### Example 3 Solving an Application Involving Finance

Serena invested money in two accounts: a savings account that yields 4.5% simple interest and a certificate of deposit (CD) that yields 7% simple interest. The amount invested at 7% was twice the amount invested at 4.5%. How much did Serena invest in each account if the total interest at the end of 1 yr was $1017.50?

**Solution:**

Let $x$ represent the amount invested in the savings account (the 4.5% account).

Let $y$ represent the amount invested in the certificate of deposit (the 7% account).

<table>
<thead>
<tr>
<th></th>
<th>4.5% Account</th>
<th>7% Account</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>$x$</td>
<td>$y$</td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>0.045$x$</td>
<td>0.07$y$</td>
<td>1017.50</td>
</tr>
</tbody>
</table>

Because the amount invested at 7% was twice the amount invested at 4.5%, we have

$$
\left( \frac{\text{Amount invested at 7%}}{\text{amount invested at 4.5%}} \right) = 2 \quad \Rightarrow \quad y = 2x
$$

From the second row of the table, we have

$$
y = 2x
$$

$$
45x + 70y = 1,017,500
$$

Multiply by 1000 to clear decimals.

Because the $y$ variable in the first equation is isolated, we will use the substitution method.

$$
45x + 70(2x) = 1,017,500
$$

Substitute the quantity $2x$ into the second equation.

$$
45x + 140x = 1,017,500
$$

Solve for $x$.

$$
185x = 1,017,500
$$

$$
x = \frac{1,017,500}{185}
$$

$$
x = 5500
$$

$$
y = 2x
$$

$$
y = 2(5500)
$$

Substitute $x = 5500$ into the equation $y = 2x$ to solve for $y$.

$$
y = 11,000
$$

Because $x = 5500$, the amount invested in the savings account is $5500.

Because $y = 11,000$, the amount invested in the CD is $11,000.

**Skill Practice**

3. Seth invested money in two accounts, one paying 5% interest and the other paying 6% interest. The amount invested at 6% was $1000 more than the amount invested at 5%. He earned a total of $830 interest in 1 yr. Use a system of equations to find the amount invested in each account.

**Answer**

3. Seth invested $7000 at 5% and $8000 at 6%.
4. Applications Involving Uniform Motion

Example 4 Solving a Distance, Rate, and Time Application

A plane flies 660 mi from Atlanta to Miami in 1.2 hr when traveling with a tailwind. The return flight against the same wind takes 1.5 hr. Find the speed of the plane in still air and the speed of the wind.

Solution:

Let \( p \) represent the speed of the plane in still air.
Let \( w \) represent the speed of the wind.

The speed of the plane with the wind:
\[
\text{Distance} = \text{Rate} \times \text{Time} \\
660 = (p + w)(1.2)
\]

The speed of the plane against the wind:
\[
660 = (p - w)(1.5)
\]

Set up a chart to organize the given information:

<table>
<thead>
<tr>
<th></th>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>With a tailwind</td>
<td>660</td>
<td>( p + w )</td>
<td>1.2</td>
</tr>
<tr>
<td>Against the wind</td>
<td>660</td>
<td>( p - w )</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Two equations can be found by using the relationship \( d = rt \).

\[
\begin{align*}
660 &= (p + w)(1.2) \\
660 &= (p - w)(1.5)
\end{align*}
\]

Notice that the first equation may be divided by 1.2 and still leave integer coefficients. Similarly, the second equation may be simplified by dividing by 1.5.
5. Applications Involving Geometry

**Example 5** Solving a Geometry Application

The sum of the two acute angles in a right triangle is 90°. The measure of one angle is 6° less than 2 times the measure of the other angle. Find the measure of each angle.

**Solution:**

Let \( x \) represent one acute angle.
Let \( y \) represent the other acute angle.

The sum of the two acute angles is 90° \( x + y = 90 \)
One angle is 6° less than 2 times the other angle \( x = 2y - 6 \)

\[
\begin{align*}
550 &= p + w \\
440 &= p - w \\
990 &= 2p & \text{Add the equations.} \\
p &= 495 \\
550 &= (495) + w & \text{Substitute } p = 495 \text{ into the equation } 550 = p + w. \\
55 &= w & \text{Solve for } w.
\end{align*}
\]

The speed of the plane in still air is 495 mph, and the speed of the wind is 55 mph.

**Skill Practice**

4. A plane flies 1200 mi from Orlando to New York in 2 hr with a tailwind. The return flight against the same wind takes 2.5 hr. Find the speed of the plane in still air and the speed of the wind.

5. Two angles are supplementary. The measure of one angle is 16° less than 3 times the measure of the other. Use a system of equations to find the measures of the angles.

**Answers**

4. The speed of the plane is 540 mph, and the speed of the wind is 60 mph.
5. The angles are 49° and 131°.
Chapter 3  Systems of Linear Equations and Inequalities

Section 3.4  Practice Exercises

Review Exercises

1. State three methods that can be used to solve a system of linear equations in two variables.

For Exercises 2–4, state which method you would prefer to use to solve the system. Then solve the system.

2. \[ y = 9 - 2x \]
   \[ 3x - y = 16 \]

3. \[ 7x - y = -25 \]
   \[ 2x + 5y = 14 \]

4. \[ 5x + 2y = 6 \]
   \[ -2x - y = 3 \]

Concept 1: Applications Involving Cost

5. A 1200-seat theater sells two types of tickets for a concert. Premium seats sell for $30 each and regular seats sell for $20 each. At one event $30,180 was collected in ticket sales with 10 seats left unsold. How many of each type of ticket was sold? (See Example 1)

6. John and Ariana bought school supplies. John spent $10.65 on 4 notebooks and 5 pens. Ariana spent $7.50 on 3 notebooks and 3 pens. What is the cost of 1 notebook and what is the cost of 1 pen?

7. Mickey bought lunch for his fellow office workers on Monday. He spent $7.35 on 3 hamburgers and 2 fish sandwiches. Chloe bought lunch on Tuesday and spent $7.15 for 4 hamburgers and 1 fish sandwich. What is the price of 1 hamburger, and what is the price of 1 fish sandwich?

8. A group of four golfers paid $150 to play a round of golf. Of the golfers, one is a member of the club and three are nonmembers. Another group of golfers consists of two members and one nonmember. They paid a total of $75. What is the cost for a member to play a round of golf, and what is the cost for a nonmember?

9. Meesha has a pocket full of change consisting of dimes and quarters. The total value is $3.15. There are 7 more quarters than dimes. How many of each coin are there?

10. Jessica has several dimes and quarters in her purse, totaling $2.70. The number of dimes is one less than the number of quarters. How many of each coin are there?

Concept 2: Applications Involving Mixtures

11. A jar of one face cream contains 18% moisturizer, and another type contains 24% moisturizer. How many ounces of each should be combined to get 12 oz of a cream that is 22% moisturizer? (See Example 2)

12. Logan wants to mix an 18% acid solution with a 45% acid solution to get 16 L of a 36% acid solution. How many liters of the 18% solution and how many liters of the 45% solution should be mixed?

13. How much fertilizer containing 8% nitrogen should be mixed with a fertilizer containing 12% nitrogen to get 8 L of a fertilizer containing 11% nitrogen?

14. How much 30% acid solution should be added to 10% acid solution to make 100 mL of a 12% acid solution?
Section 3.4 Applications of Systems of Linear Equations in Two Variables

15. How much pure bleach should Tim combine with a solution that is 4% bleach to make 12 oz of a 12% bleach solution?

16. A fruit punch that contains 25% fruit juice is combined with 100% fruit juice. How many ounces of each should be used to make 48 oz of a mixture that is 75% fruit juice?

Concept 3: Applications Involving Principal and Interest

17. Mr. Coté invested 3 times as much money in a stock fund that returned 8% interest after 1 yr as he did in a bond fund that earned 5% interest. If his total earnings came to $435 after 1 yr, how much did he invest in each fund? (See Example 3.)

18. Aliya deposited half as much money in a savings account earning 2.5% simple interest as she invested in a money market account that earns 3.5% simple interest. If the total interest after 1 yr is $247, how much did she invest in each account?

19. A credit union offers 5.5% simple interest on a certificate of deposit (CD) and 3.5% simple interest on a savings account. If Mr. Levy invested $200 more in the CD than in the savings account and the total interest after the first year was $245, how much was invested in each account?

20. Jody invested $5000 less in an account paying 3% simple interest than she did in an account paying 4% simple interest. At the end of the first year, the total interest from both accounts was $725. Find the amount invested in each account.

21. Alina borrowed a total of $15,000 from two banks to buy a new boat. Because of her excellent credit, one bank charged only 6% simple interest and the other charged 7% simple interest. At the end of 5 yr, the total amount of money she paid in interest was $4750. How much did she borrow from each bank?

22. Didi plans to take a trip to the Galapagos Islands in 4 yr and knows that she needs approximately $3500 for the trip. She invests a total of $15,500 in two funds. One fund has a 6% return, and the other has a 5% return. How much should she invest in each fund so that she earns $3500 after 4 yr?

Concept 4: Applications Involving Uniform Motion

23. It takes a boat 2 hr to travel 16 mi downstream with the current and 4 hr to return against the current. Find the speed of the boat in still water and the speed of the current. (See Example 4.)

24. A plane flew 720 mi in 3 hr with the wind. It would take 4 hr to travel the same distance against the wind. What is the speed of the plane in still air and the speed of the wind?

25. A plane flies from Atlanta to Los Angeles against the wind in 5 hr. The return trip back to Atlanta with the wind takes only 4 hr. If the distance between Atlanta and Los Angeles is 3200 km, find the speed of the plane in still air and the speed of the wind.

26. The Gulf Stream is a warm ocean current that extends from the eastern side of the Gulf of Mexico up through the Florida Straits and along the southeastern coast of the United States to Cape Hatteras, North Carolina. A boat travels with the current 100 mi from Miami, Florida, to Freeport, Bahamas, in 2.5 hr. The return trip against the same current takes $\frac{3}{2}$ hr. Find the speed of the boat in still water and the speed of the current.

27. A moving sidewalk in the Atlanta airport moves people between gates. It takes Molly’s 8-year-old son Stephen 20 sec to travel 100 ft walking with the sidewalk. It takes him 30 sec to travel 60 ft walking against the moving sidewalk (in the opposite direction). Find the speed of the moving sidewalk and Stephen’s walking speed on nonmoving ground.

28. Kim rides a total of 48 km in the bicycle portion of a triathlon. The course is an “out and back” route. It takes her 3 hr on the way out against the wind. The ride back takes her 2 hr with the wind. Find the speed of the wind and Kim’s speed riding her bike in still air.
Concept 5: Applications Involving Geometry

For Exercises 29–34, solve the applications involving geometry. If necessary, refer to the geometry formulas listed in the inside back cover of the text.

29. In a right triangle, one acute angle measures 6° more than 3 times the other. If the sum of the measures of the two acute angles must equal 90°, find the measures of the acute angles. (See Example 5.)

30. An isosceles triangle has two angles of the same measure. If the angle represented by y measures 3° less than the angle x, find the measures of all angles of the triangle.

31. Two angles are supplementary. One angle measures 2° less than 3 times the other. What are the measures of the two angles?

32. The measure of one angle is 5 times the measure of another. If the two angles are supplementary, find the measures of the angles.

33. One angle measures 6° more than twice another. If the two angles are complementary, find the measures of the angles.

34. Two angles are complementary. One angle measures 15° more than 2 times the measure of the other. What are the measures of the two angles?

Mixed Exercises

35. How much pure gold (24K) must be mixed with 60% gold to get 20 grams (g) of 75% gold?

36. Connie is the head of maintenance at a large hospital. She received news of a new state mandate indicating that the minimum strength for disinfectant was to be 17%, up from the old requirement of 15%. Connie had plenty of barrels of 15% disinfectant left over, and also lots of the strong 55% disinfectant used in rooms for patients with highly contagious diseases. How many gallons of each disinfectant should be mixed to get 50 gal of 17% disinfectant?

37. A rowing team trains on the Halifax River. It can row upstream 10 mi against the current in 2.5 hr and 16 mi downstream with the current in the same amount of time. Find the speed of the boat in still water and the speed of the current.

38. In her kayak, Bonnie can travel 31.5 mi downstream with the current in 7 hr. The return trip against the current takes 9 hr. Find the speed of the kayak in still water and the speed of the current.

39. There are two types of tickets sold at the Canadian Formula One Grand Prix race. The price of 6 grandstand tickets and 2 general admissions tickets costs $2330. The price of 4 grandstand tickets and 4 general admission tickets cost $2020. What is the price of each type of ticket?

40. A basketball player scored 19 points by shooting two-point and three-point baskets. If she made a total of eight baskets, how many of each type did she make?
Section 3.5 Linear Inequalities and Systems of Linear Inequalities in Two Variables

41. A bank offers two accounts, a money market account at 2% simple interest and a regular savings account at 1.3% interest. If Svetlana deposits $3000 between the two accounts and receives $51.25 in total interest in the first year, how much did she invest in each account?

42. Angelo invested $8000 in two accounts: one that pays 3% and one that pays 1.8%. At the end of the first year, his total interest earned was $222. How much did he deposit in the account that pays 3%?

43. The perimeter of a rectangle is 42 m. The length is 1 m longer than the width. Find the dimensions of the rectangle.

44. In a right triangle, the measure of one acute angle is one-fourth the measure of the other. Find the measures of the acute angles.

45. A coin collection consists of 50¢ pieces and $1 coins. If there are 21 coins worth $15.50, how many 50¢ pieces and $1 coins are there?

46. Jacob has a piggy bank consisting of nickels and dimes. If there are 30 coins worth $1.90, how many nickels and dimes are in the bank?

47. One phone company charges $0.15 per minute for long-distance calls. A second company charges only $0.10 per minute for long-distance calls, but adds a monthly fee of $4.95.
   a. Write a linear function representing the cost for the first company for x minutes.
   b. Write a linear function representing the cost for the second company for x minutes.
   c. Find the number of minutes of long-distance calling for which the total bill from either company would be the same.

48. A rental car company rents a compact car for $20 a day, plus $0.25 per mile. A midsize car rents for $30 a day, plus $0.20 per mile.
   a. Write a linear function representing the cost to rent the compact car for x miles.
   b. Write a linear function representing the cost to rent a midsize car for x miles.
   c. Find the number of miles at which the cost to rent either car would be the same.

1. Graphing Linear Inequalities in Two Variables

A linear inequality in two variables \( x \) and \( y \) is an inequality that can be written in one of the following forms: \( ax + by < c \), \( ax + by > c \), \( ax + by \leq c \), or \( ax + by \geq c \), provided \( a \) and \( b \) are not both zero.

A solution to a linear inequality in two variables is an ordered pair that makes the inequality true. For example, solutions to the inequality \( x + y < 6 \) are ordered pairs \( (x, y) \) such that the sum of the \( x \)- and \( y \)-coordinates is less than 6. This inequality has an infinite number of solutions, and therefore it is convenient to express the solution set as a graph.

To graph a linear inequality in two variables, we will follow these steps.
Chapter 3 Systems of Linear Equations and Inequalities

PROCEDURE Graphing a Linear Inequality in Two Variables

**Step 1** Solve for \( y \), if possible.

**Step 2** Graph the related equation. Draw a dashed line if the inequality is strict, \(<\) or \(><\). Otherwise, draw a solid line.

**Step 3** Shade above or below the line as follows:
- Shade above the line if the inequality is of the form \( y > ax + b \) or \( y \geq ax + b \).
- Shade below the line if the inequality is of the form \( y < ax + b \) or \( y \leq ax + b \).

*Note:* A dashed line indicates that the line is not included in the solution set. A solid line indicates that the line is included in the solution set.

This process is demonstrated in Example 1.

**Example 1** Graphing a Linear Inequality in Two Variables

Graph the solution set. \(-3x + y \leq 1\)

**Solution:**

\[-3x + y \leq 1\]

\[y \leq 3x + 1\]  Solve for \( y \).

Next graph the line defined by the related equation \( y = 3x + 1 \).

Because the inequality is of the form \( y \leq ax + b \), the solution to the inequality is the region **below** the line \( y = 3x + 1 \). See Figure 3-9.

**Skill Practice** Graph the solution set.

1. \(2x + y \geq -4\)

After graphing the solution to a linear inequality, we can verify that we have shaded the correct side of the line by using test points. In Example 1, we can pick an arbitrary ordered pair within the shaded region. Then substitute the \( x \)- and \( y \)-coordinates in the original inequality. If the result is a true statement, then that ordered pair is a solution to the inequality and suggests that other points from the same region are also solutions.

For example, the point \((0, 0)\) lies within the shaded region (Figure 3-10).

\[-3x + y \leq 1\]  Substitute \((0, 0)\) in the original inequality.

\[-3(0) + (0) \leq 1\]

\[0 + 0 \leq 1\] ✔ True  The point \((0, 0)\) from the shaded region is a solution.
Section 3.5  Linear Inequalities and Systems of Linear Inequalities in Two Variables

In Example 2, we will graph the solution set to a strict inequality. A strict inequality uses the symbol < or >. In such a case, the boundary line will be drawn as a dashed line. This indicates that the boundary itself is not part of the solution set.

**Example 2**  Graphing a Linear Inequality in Two Variables

Graph the solution set.  

\[-4y < 5x\]

**Solution:**

\[-4y < 5x\]

\[-4y > 5x\]

\[-\frac{4}{-4} > \frac{5x}{-4}\]

\[y > -\frac{5}{4}x\]

Solve for \(y\). Because we divide both sides by a negative number, reverse the inequality sign.

Graph the line defined by the related equation, \(y = -\frac{5}{4}x\). The boundary line is drawn as a dashed line because the inequality is strict. Also note that the line passes through the origin.

Because the inequality is of the form \(y > ax + b\), the solution to the inequality is the region above the line. See Figure 3-11.

**Skill Practice**  Graph the solution set.

2. \(-3y < x\)

In Example 2, we cannot use the origin as a test point, because the point \((0, 0)\) is on the boundary line. Be sure to select a test point strictly within the shaded region. In this case, we choose \((2, 1)\). See Figure 3-12.

\[-4y < 5x\]

\[-4(1) < 5(2)\]

\[-4 < 10\]  True  

Substitute \((2, 1)\) in the original inequality. The point \((2, 1)\) from the shaded region is a solution to the original inequality.

In Example 3, we encounter a situation in which we cannot solve for the \(y\) variable.
Chapter 3 Systems of Linear Equations and Inequalities

Graphing a Linear Inequality

Graph the solution set. \(4x \geq -12\)

Solution:

\[4x \geq -12\]
\[x \geq -3\]

In this inequality, there is no \(y\) variable. However, we can simplify the inequality by solving for \(x\).

Graph the related equation \(x = -3\). This is a vertical line. The boundary is drawn as a solid line, because the inequality is not strict, \(\geq\).

To shade the appropriate region, refer to the inequality, \(x \geq -3\). The points for which \(x\) is greater than \(-3\) are to the right of \(x = -3\). Therefore, shade the region to the right of the line (Figure 3-13).

Selecting a test point such as \((0, 0)\) from the shaded region indicates that we have shaded the correct side of the line.

Example 3 Graphing a Linear Inequality

\[\frac{4}{4}(0) \geq -12\]  Substitute \(x = 0\).

\[4(0) \geq -12 \checkmark\] True

Skill Practice Graph the solution set.

3. \(-2x \geq 2\)

2. Compound Linear Inequalities in Two Variables

Some applications require us to find the union or intersection of the solution sets of two or more linear inequalities.

Example 4 Graphing a Compound Linear Inequality

Graph the solution set of the compound inequality.

\[y > \frac{1}{2}x + 1\] and \[x + y < 1\]

Solution:

Solve each inequality for \(y\).

First inequality

\[y > \frac{1}{2}x + 1\]

Second inequality

\[x + y < 1\]

\[y < -x + 1\]

The inequality is of the form \(y > ax + b\). Shade above the boundary line. (See Figure 3-14.)

The inequality is of the form \(y < ax + b\). Shade below the boundary line. (See Figure 3-15.)
The region bounded by the inequalities is the region above the line \( y = \frac{3}{2}x + 1 \) and below the line \( y = -x + 1 \). This is the intersection or “overlap” of the two regions (shown in purple in Figure 3-16).

The intersection is the solution set to the system of inequalities. See Figure 3-17.

**Skill Practice** Graph the solution set.

4. \( x - 3y > 3 \) and \( y < -2x + 4 \)

Example 5 demonstrates the union of the solution sets of two linear inequalities.

**Example 5** **Graphing a Compound Linear Inequality**

Graph the solution set of the compound inequality.

\[
3y \leq 6 \quad \text{or} \quad y - x \leq 0
\]

**Solution:**

**First inequality**

\[
3y \leq 6
\]

\[
y \leq 2
\]

The graph of \( y \leq 2 \) is the region on and below the horizontal line \( y = 2 \). (See Figure 3-18.)

**Second inequality**

\[
y - x \leq 0
\]

\[
y \leq x
\]

The inequality \( y \leq x \) is of the form \( y \leq ax + b \). Graph a solid line and the region below the line. (See Figure 3-19.)

**Answer**

4.
### Example 6 Graphing a Compound Linear Inequality

Describe the region of the plane defined by the system of inequalities.

\[ x \leq 0 \quad \text{and} \quad y \geq 0 \]

**Solution:**

- \( x \leq 0 \) on the \( y \)-axis and in the second and third quadrants.
- \( y \geq 0 \) on the \( x \)-axis and in the first and second quadrants.

The intersection of these regions is the set of points in the second quadrant (with the boundary included).

**Skill Practice** Graph the region defined by the system of inequalities.

5. \( 2y \leq 4 \quad \text{or} \quad y \leq x + 1 \)

### 3. Graphing a Feasible Region

When two variables are related under certain constraints, a system of linear inequalities can be used to show a region of feasible values for the variables. The *feasible region* represents the ordered pairs that are true for each inequality in the system.

**Example 7** Graphing a Feasible Region

Susan has two tests on Friday: one in chemistry and one in psychology. Because the two classes meet in consecutive hours, she has no study time between tests. Susan estimates that she has a maximum of 12 hr of study time before the tests, and she must divide her time between chemistry and psychology.

Let \( x \) represent the number of hours Susan spends studying chemistry.

Let \( y \) represent the number of hours Susan spends studying psychology.

a. Find a set of inequalities to describe the constraints on Susan’s study time.

b. Graph the constraints to find the feasible region defining Susan’s study time.
Solution:
a. Because Susan cannot study chemistry or psychology for a negative period of time, we have \( x \geq 0 \) and \( y \geq 0 \). Furthermore, her total time studying cannot exceed 12 hr: \( x + y \leq 12 \).

A system of inequalities that defines the constraints on Susan's study time is:

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
x + y & \leq 12
\end{align*}
\]

b. The first two conditions \( x \geq 0 \) and \( y \geq 0 \) represent the set of points in the first quadrant. The third condition \( x + y \leq 12 \) represents the set of points below and including the line \( x + y = 12 \) (Figure 3-21).

Discussion:

1. Refer to the feasible region. Is the ordered pair (8, 5) part of the feasible region?
   No. The ordered pair (8, 5) indicates that Susan spent 8 hr studying chemistry and 5 hr studying psychology. This is a total of 13 hr, which exceeds the constraint that Susan only had 12 hr to study. The point (8, 5) lies outside the feasible region, above the line \( x + y = 12 \) (Figure 3-22).

2. Is the ordered pair (7, 3) part of the feasible region?
   Yes. The ordered pair (7, 3) indicates that Susan spent 7 hr studying chemistry and 3 hr studying psychology. This point lies within the feasible region and satisfies all three constraints.

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
x + y & \leq 12
\end{align*}
\]

3. Suppose there was one additional constraint imposed on Susan's study time. She knows she needs to spend at least twice as much time studying chemistry as she does studying psychology.
   Because the time studying chemistry must be at least twice the time studying psychology, we have \( x \geq 2y \).

This inequality may also be written as \( y \leq \frac{1}{2}x \).

Figure 3-23 shows the first quadrant with the constraint \( y \leq \frac{1}{2}x \).
Chapter 3 Systems of Linear Equations and Inequalities

4. At what point in the feasible region is Susan making the most efficient use of her time for both classes?

First and foremost, Susan must make use of all 12 hr. This occurs for points along the line \( x + y = 12 \). Susan will also want to study for both classes with approximately twice as much time devoted to chemistry. Therefore, Susan will be deriving the maximum benefit at the point of intersection of the line \( x + y = 12 \) and the line \( y = \frac{1}{2}x \).

Using the substitution method, replace \( y = \frac{1}{2}x \) into the equation \( x + y = 12 \).

\[
x + \frac{1}{2}x = 12
\]

Clear fractions.

\[
2x + x = 24
\]

Solve for \( x \).

\[
x = 8
\]

To solve for \( y \), substitute \( x = 8 \) into the equation \( y = \frac{1}{2}x \).

\[
y = \frac{8}{2} = 4
\]

Therefore, Susan should spend 8 hr studying chemistry and 4 hr studying psychology.

Skill Practice

7. A local pet rescue group has a total of 30 cages that can be used to hold cats and dogs. Let \( x \) represent the number of cages used for cats, and let \( y \) represent the number used for dogs.

a. Write a set of inequalities to express the fact that the number of cat and dog cages cannot be negative.

b. Write an inequality to describe the constraint on the total number of cages for cats and dogs.

c. Graph the system of inequalities to find the feasible region describing the available cages.

Section 3.5 Practice Exercises

Study Skills Exercise

1. Define the key term linear inequality in two variables.

Review Exercises

For Exercises 2–5, solve the inequalities.

2. \( 5 < x + 1 \) and \( -2x + 6 \geq -6 \)

3. \( 5 - x \leq 4 \) and \( 6 > 3x - 3 \)

4. \( 4 - y < 3y + 12 \) or \( -2(y + 3) \geq 12 \)

5. \( -2x < 4 \) or \( 3x - 1 \leq -13 \)
Concept 1: Graphing Linear Inequalities in Two Variables

For Exercises 6–9, decide if the given point is a solution to the inequality.

6. $2x - y > 8$
   a. (3, −5)  
   b. (−1, −10)
   c. (4, −2)  
   d. (0, 0)

7. $3y + x < 5$
   a. (−1, 7)  
   b. (5, 0) 
   c. (0, 0)  
   d. (2, −3)

8. $y \leq -2$
   a. (5, −3)  
   b. (−4, −2)
   c. (0, 0)  
   d. (3, 2)

9. $x \geq 5$
   a. (4, 5)  
   b. (5, −1)
   c. (8, 8)  
   d. (0, 0)

10. When should you use a dashed line to graph the solution to a linear inequality?

For Exercises 11–16, decide which inequality symbol should be used ($<$, $>$, $\geq$, $\leq$) by looking at the graph.

11. $x - y \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad
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20.  $x - 3y < 8$

21.  $2x \leq -6y + 12$

22.  $4x < 3y + 12$

23.  $2y \leq 4x$

24.  $-6x < 2y$

25.  $y \geq -2$

26.  $y \geq 5$

27.  $4x < 5$

28.  $x + 6 < 7$

29.  $y \geq \frac{2}{5}x - 4$

30.  $y \geq \frac{-5}{2}x - 4$

31.  $y \leq \frac{1}{3}x + 6$
Section 3.5  Linear Inequalities and Systems of Linear Inequalities in Two Variables

32.  \( y \leq -\frac{1}{4}x + 2 \)  

33.  \( y - 5x > 0 \)  

34.  \( y - \frac{1}{2}x > 0 \)  

35.  \( \frac{x}{5} + \frac{y}{4} < 1 \)  

36.  \( x + \frac{y}{2} \geq 2 \)  

37.  \( 0.1x + 0.2y \leq 0.6 \)  

38.  \( 0.3x - 0.2y < 0.6 \)  

39.  \( x \leq \frac{2}{3}y \)  

40.  \( x \geq \frac{5}{4} \)  

Concept 2: Compound Linear Inequalities in Two Variables

For Exercises 41–55, graph the solution set of each compound inequality.  (See Examples 4–6.)

41. \( y < 4 \) and \( y > -x + 2 \)  

42. \( y < 3 \) and \( x + 2y < 6 \)  

43. \( 2x + y \leq 5 \) or \( x \geq 3 \)
Chapter 3  Systems of Linear Equations and Inequalities

44. \( x + 3y \geq 3 \) or \( x \leq -2 \)

45. \( x + y < 3 \) and \( 4x + y \leq 6 \)

46. \( x + y < 4 \) and \( 3x + y < 9 \)

47. \( 2x - y \leq 2 \) or \( 2x + 3y \geq 6 \)

48. \( 3x + 2y \geq 4 \) or \( x - y \leq 3 \)

49. \( x > 4 \) and \( y < 2 \)

50. \( x < 3 \) and \( y > 4 \)

51. \( x \leq -2 \) or \( y \leq 0 \)

52. \( x \geq 0 \) or \( y \geq -3 \)

53. \( x > 0 \) and \( x + y < 6 \)

54. \( x < 0 \) and \( x + y < 2 \)

55. \( y \leq 0 \) or \( x - y \leq -4 \)
Concept 3: Graphing a Feasible Region

For Exercises 56–59, graph the feasible regions.

56. \( x + y \leq 3 \) and \( x \geq 0, y \geq 0 \)
57. \( x - y \leq 2 \) and \( x \geq 0, y \geq 0 \)
58. \( x \geq 0, y \geq 0 \)
59. \( x + y \leq 8 \) and \( 3x + 5y \leq 30 \)

60. Suppose Sue has 50 ft of fencing with which she can build a rectangular dog run. Let \( x \) represent the length of the dog run and let \( y \) represent the width.
   a. Write an inequality representing the fact that the total perimeter of the dog run is at most 50 ft.
   b. Sketch part of the solution set for this inequality that represents all possible values for the length and width of the dog run. (Hint: Note that both the length and the width must be positive.)

61. Suppose Rick has 40 ft of fencing with which he can build a rectangular garden. Let \( x \) represent the length of the garden and let \( y \) represent the width.
   a. Write an inequality representing the fact that the total perimeter of the garden is at most 40 ft.
   b. Sketch part of the solution set for this inequality that represents all possible values for the length and width of the garden. (Hint: Note that both the length and the width must be positive.)

62. A manufacturer produces two models of desks. Model A requires 4 hr to stain and finish and 3 hr to assemble. Model B requires 3 hr to stain and finish and 1 hr to assemble. The total amount of time available for staining and finishing is 24 hr and for assembling is 12 hr. Let \( x \) represent the number of Model A desks produced, and let \( y \) represent the number of Model B desks produced.
   a. Write two inequalities that express the fact that the number of desks to be produced cannot be negative.
   b. Write an inequality in terms of the number of Model A and Model B desks that can be produced if the total time for staining and finishing is at most 24 hr.
   c. Write an inequality in terms of the number of Model A and Model B desks that can be produced if the total time for assembly is no more than 12 hr.
   d. Graph the feasible region formed by graphing the preceding inequalities.
   e. Is the point \((3, 1)\) in the feasible region? What does the point \((3, 1)\) represent in the context of this problem?
   f. Is the point \((5, 4)\) in the feasible region? What does the point \((5, 4)\) represent in the context of this problem?
63. In scheduling two drivers for delivering pizza, James needs to have at least 65 hr scheduled this week. His two drivers, Karen and Todd, are not allowed to get overtime, so each one can work at most 40 hr. Let \( x \) represent the number of hours that Karen can be scheduled, and let \( y \) represent the number of hours Todd can be scheduled. (See Example 7.)

a. Write two inequalities that express the fact that Karen and Todd cannot work a negative number of hours.

b. Write two inequalities that express the fact that neither Karen nor Todd is allowed overtime (i.e., each driver can have at most 40 hr).

c. Write an inequality that expresses the fact that the total number of hours from both Karen and Todd needs to be at least 65 hr.

d. Graph the feasible region formed by graphing the inequalities.

e. Is the point (35, 40) in the feasible region? What does the point (35, 40) represent in the context of this problem?

f. Is the point (20, 40) in the feasible region? What does the point (20, 40) represent in the context of this problem?

---

### Section 3.6 Systems of Linear Equations in Three Variables and Applications

#### Concepts

1. Solutions to Systems of Linear Equations in Three Variables
2. Solving Systems of Linear Equations in Three Variables
3. Applications of Linear Equations in Three Variables
4. Solving Dependent and Inconsistent Systems

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#### 1. Solutions to Systems of Linear Equations in Three Variables

In Sections 3.1–3.3, we solved systems of linear equations in two variables. In this section, we will expand the discussion to solving systems involving three variables.

A **linear equation in three variables** can be written in the form \( Ax + By + Cz = D \), where \( A, B, \) and \( C \) are not all zero. For example, the equation \( 2x + 3y + z = 6 \) is a linear equation in three variables. Solutions to this equation are **ordered triples** of the form \((x, y, z)\) that satisfy the equation. Some solutions to the equation \( 2x + 3y + z = 6 \) are:

- Solution: \((1, 1, 1)\) → \(2(1) + 3(1) + (1) = 6\) ✔ True
- Solution: \((0, 1, 3)\) → \(2(0) + 3(1) + (3) = 6\) ✔ True
- Solution: \((2, 0, 2)\) → \(2(2) + 3(0) + (2) = 6\) ✔ True

Infinitely many ordered triples serve as solutions to the equation \( 2x + 3y + z = 6 \).

The set of all ordered triples that are solutions to a linear equation in three variables may be represented graphically by a plane in space. Figure 3-24 shows a portion of the plane \( 2x + 3y + z = 6 \) in a 3-dimensional coordinate system.

An example of a system of three linear equations in three variables is shown here.

\[
\begin{align*}
2x + y - 3z &= -7 \\
3x - 2y + z &= 11 \\
-2x - 3y - 2z &= 3
\end{align*}
\]
A solution to a system of linear equations in three variables is an ordered triple that satisfies each equation. Geometrically, a solution is a point of intersection of the planes represented by the equations in the system.

A system of linear equations in three variables may have one unique solution, infinitely many solutions, or no solution (Table 3-2, Table 3-3, and Table 3-4).

**Table 3-2**

One unique solution (planes intersect at one point)
- The system is consistent.
- The system is independent.

**Table 3-3**

No solution (the three planes do not all intersect)
- The system is inconsistent.
- The system is independent.

**Table 3-4**

Infinitely many solutions (planes intersect at infinitely many points)
- The system is consistent.
- The system is dependent.
2. Solving Systems of Linear Equations in Three Variables

To solve a system involving three variables, the goal is to eliminate one variable. This reduces the system to two equations in two variables. One strategy for eliminating a variable is to pair up the original equations two at a time.

**PROCEDURE Solving a System of Three Linear Equations in Three Variables**

**Step 1** Write each equation in standard form $Ax + By + Cz = D$.

**Step 2** Choose a pair of equations, and eliminate one of the variables by using the addition method.

**Step 3** Choose a different pair of equations and eliminate the same variable.

**Step 4** Once steps 2 and 3 are complete, you should have two equations in two variables. Solve this system by using the methods from Sections 3.2 and 3.3.

**Step 5** Substitute the values of the variables found in step 4 into any of the three original equations that contain the third variable. Solve for the third variable.

**Step 6** Check the ordered triple in each of the original equations. Then write the solution as an ordered triple within set notation.

### Example 1 Solving a System of Linear Equations in Three Variables

Solve the system.

$$\begin{align*}
2x + y - 3z &= -7 \\
3x - 2y + z &= 11 \\
-2x - 3y - 2z &= 3
\end{align*}$$

**Solution:**

<table>
<thead>
<tr>
<th>A</th>
<th>$2x + y - 3z = -7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$3x - 2y + z = 11$</td>
</tr>
<tr>
<td>C</td>
<td>$-2x - 3y - 2z = 3$</td>
</tr>
</tbody>
</table>

**Step 1:** The equations are already in standard form.

- It is often helpful to label the equations.
- The $y$ variable can be easily eliminated from equations A and B and from equations A and C. This is accomplished by creating opposite coefficients for the $y$ terms and then adding the equations.

**Step 2:** Eliminate the $y$ variable from equations A and B.

<table>
<thead>
<tr>
<th>A</th>
<th>$2x + y - 3z = -7$</th>
<th>Multiply by 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$3x - 2y + z = 11$</td>
<td>$4x + 2y - 6z = -14$</td>
</tr>
</tbody>
</table>

$7x - 5z = -3$
Step 3: Eliminate the y variable again, this time from equations \( \text{A} \) and \( \text{C} \).

\[
\begin{align*}
\text{A} & : 2x + y - 3z = -7 \\
\text{C} & : -2x + 3y - 2z = 3 \\
\end{align*}
\]

Multiply by 3:

\[
\begin{align*}
6x + 3y - 9z & = -21 \\
-2x - 3y - 2z & = 3 \\
4x & = -11z = -18 \quad \text{[E]}
\end{align*}
\]

Step 4: Now equations \( \text{D} \) and \( \text{E} \) can be paired up to form a linear system in two variables. Solve this system.

\[
\begin{align*}
\text{D} & : 7x - 5z = -3 \quad \text{Multiply by 4} \\
\text{E} & : 4x - 11z = -18 \quad \text{Multiply by 7}
\end{align*}
\]

\[
\begin{align*}
28x + 20z & = 12 \\
28x - 77z & = -126 \\
-57z & = -114 \\
\text{z} & = 2
\end{align*}
\]

Once one variable has been found, substitute this value into either equation in the two-variable system, that is, either equation \( \text{D} \) or \( \text{E} \).

\[
\begin{align*}
\text{D} & : 7x - 5(2) = -3 \\
7x - 10 & = -3 \\
7x & = 7 \\
x & = 1
\end{align*}
\]

\[
\begin{align*}
\text{A} & : 2x + y - 3z = -7 \\
2(1) + y - 3(2) & = -7 \\
2 + y - 6 & = -7 \\
y - 4 & = -7 \\
y & = -3
\end{align*}
\]

Step 5: Now that two variables are known, substitute these values \((x \text{ and } z)\) into any of the original three equations to find the remaining variable \(y\).

Substitute \(x = 1\) and \(z = 2\) into equation \( \text{A} \).

\[
\begin{align*}
2x + y - 3z & = -7 \\
2(1) + (-3) - 3(2) & = -7 \quad \text{True}
\end{align*}
\]

The solution set is \(((1, -3, 2))\).

Step 6: Check the ordered triple in the three original equations.

\[
\begin{align*}
\text{Check:} & \\
2x + y - 3z & = -7 \\
3x - 2y + z & = 11 \\
-2x - 3y - 2z & = 3
\end{align*}
\]

- \(2(1) + (-3) - 3(2) = -7 \quad \text{True}\)
- \(3(1) - 2(-3) + (2) = 11 \quad \text{True}\)
- \(-2(1) - 3(-3) - 2(2) = 3 \quad \text{True}\)

Skill Practice: Solve the system.

1. \(x + 2y - z = 1\)  
   \(3x - y + 2z = 13\)  
   \(2x + 3y - z = -8\)

Answer

1. \(((1, -2, 4))\)
3. Applications of Linear Equations in Three Variables

Example 2 Applying Systems of Linear Equations to Geometry

In a triangle, the smallest angle measures 10° more than one-half the measure of the largest angle. The middle angle measures 12° more than the measure of the smallest angle. Find the measure of each angle.

Solution:

Let \( x \) represent the measure of the smallest angle.
Let \( y \) represent the measure of the middle angle.
Let \( z \) represent the measure of the largest angle.

To solve for three variables, we need to establish three independent relationships among \( x \), \( y \), and \( z \).

\[
\begin{align*}
A & : \quad x = \frac{z}{2} + 10 \\
B & : \quad y = x + 12 \\
C & : \quad x + y + z = 180
\end{align*}
\]

The smallest angle measures 10° more than one-half the measure of the largest angle.
The middle angle measures 12° more than the measure of the smallest angle.
The sum of the angles inscribed in a triangle is 180°.

Clear fractions and write each equation in standard form.

\[
\begin{align*}
A & : \quad x = \frac{z}{2} + 10 \quad \text{Multiply by } 2 \quad 2x = z + 20 \quad 2x - z = 20 \\
B & : \quad y = x + 12 \quad -x + y = 12 \\
C & : \quad x + y + z = 180 \quad x + y + z = 180
\end{align*}
\]

Notice equation \( B \) is missing the \( z \) variable. Therefore, we can eliminate \( z \) again by pairing up equations \( A \) and \( C \).

\[
\begin{align*}
A & : \quad 2x - z = 20 \\
C & : \quad x + y + z = 180 \quad 3x + y = 200 \\
& \quad \underline{\text{Multiply by } -1} \quad -x + y = -12 \\
D & : \quad 3x + y = 200 \quad 3x + y = 200 \quad 3x + y = 200 \quad \underline{\text{Multiply by } -1} \quad 4x = 188 \\
\end{align*}
\]

\[
\begin{align*}
D & : \quad 4x = 188 \quad x = 47
\end{align*}
\]

Solve for \( x \).

From equation \( B \) we have \(-x + y = 12\) \(\Rightarrow -47 + y = 12 \Rightarrow y = 59\)
From equation \( C \) we have \(x + y + z = 180\) \(\Rightarrow 47 + 59 + z = 180 \Rightarrow z = 74\)

The smallest angle measures 47°, the middle angle measures 59°, and the largest angle measures 74°.
Example 3  Applying Systems of Linear Equations to Nutrition

Doctors have become increasingly concerned about the sodium intake in the U.S. diet. Recommendations by the American Medical Association indicate that most individuals should not exceed 2400 mg of sodium per day.

Torie ate 1 slice of pizza, 1 serving of ice cream, and 1 glass of soda for a total of 1030 mg of sodium. David ate 3 slices of pizza, no ice cream, and 2 glasses of soda for a total of 2420 mg of sodium. Emilie ate 2 slices of pizza, 1 serving of ice cream, and 2 glasses of soda for a total of 1910 mg of sodium. How much sodium is in one serving of each item?

Solution:

Let \( x \) represent the sodium content of 1 slice of pizza.
Let \( y \) represent the sodium content of 1 serving of ice cream.
Let \( z \) represent the sodium content of 1 glass of soda.

From Torie’s meal we have:\n\[ A \quad x + y + z = 1030 \]
From David’s meal we have: \[ B \quad 3x + 2z = 2420 \]
From Emilie’s meal we have: \[ C \quad 2x + y + 2z = 1910 \]
Equation \( B \) is missing the \( y \) variable. Eliminating \( y \) from equations \( A \) and \( C \), we have
\[ A \quad x + y + z = 1030 \quad \text{Multiply by -1:} \quad -x - y - z = -1030 \]
\[ C \quad 2x + y + 2z = 1910 \quad \text{Multiply by 2:} \quad -2x - 2y - 4z = -3860 \]

Solve the system formed by equations \( B \) and \( D \).
\[ B \quad 3x + 2z = 2420 \quad \text{Multiply by 2:} \quad 6x + 4z = 4840 \]
\[ D \quad x + z = 880 \quad \text{Subtract:} \quad -5x = -400 \]
\[ x = 660 \]

From equation \( D \) we have \( x + z = 880 \quad \rightarrow 660 + z = 880 \quad \rightarrow z = 220 \)
From equation \( A \) we have \( x + y + z = 1030 \quad \rightarrow 660 + y + 220 = 1030 \quad \rightarrow y = 150 \)

Therefore, 1 slice of pizza has 660 mg of sodium, 1 serving of ice cream has 150 mg of sodium, and 1 glass of soda has 220 mg of sodium.

Skill Practice

3. Annette, Barb, and Carlita work in a clothing shop. One day the three had combined sales of $1480. Annette sold $120 more than Barb. Barb and Carlita combined sold $280 more than Annette. How much did each person sell?

Answers

2. The sides are 8 in., 10 in., and 12 in.
3. Annette sold $600, Barb sold $480, and Carlita sold $400.
Chapter 3 Systems of Linear Equations and Inequalities

4. Solving Dependent and Inconsistent Systems

**Example 4** Solving a Dependent System of Linear Equations

Solve the system. If there is not a unique solution, label the system as either dependent or inconsistent.

\[ \begin{align*}
A & : 3x + y - z = 8 \\
B & : 2x - y + 2z = 3 \\
C & : x + 2y - 3z = 5
\end{align*} \]

**Solution:**

The first step is to make a decision regarding the variable to eliminate. The \( y \) variable is particularly easy to eliminate because the coefficients of \( y \) in equations \( A \) and \( B \) are already opposites. The \( y \) variable can be eliminated from equations \( B \) and \( C \) by multiplying equation \( B \) by 2.

\[ \begin{align*}
A & : 3x + y - z = 8 \\
B & : 2x - y + 2z = 3 \\
C & : x + 2y - 3z = 5 \\
\text{Pair up equations } A \text{ and } B \text{ to eliminate } y.
\end{align*} \]

\[ \begin{align*}
B & : 2x - y + 2z = 3 \quad \text{Multiply by 2} \\
\text{Pair up equations } B \text{ and } C \text{ to eliminate } y.
\end{align*} \]

\[ \begin{align*}
C & : x + 2y - 3z = 5 \quad \text{Multiply by 5} \\
\text{Pair up equations } B \text{ and } C \text{ to eliminate } y.
\end{align*} \]

Because equations \( D \) and \( E \) are equivalent equations, it appears that this is a dependent system. By eliminating variables we obtain the identity \( 0 = 0 \).

\[ \begin{align*}
D & : 5x + z = 11 \quad \text{Multiply by -1} \\
E & : 5x + z = 11 \\
\text{The result } 0 = 0 \text{ indicates that there are infinitely many solutions and that the system is dependent.}
\end{align*} \]

**Skill Practice** Solve the system. If the system does not have a unique solution, identify the system as dependent or inconsistent.

4. \[ \begin{align*}
x + y + z &= 8 \\
2x - y + z &= 6 \\
-5x - 2y - 4z &= -30
\end{align*} \]

**Answer**

4. Infinitely many solutions; dependent system
Section 3.6 Systems of Linear Equations in Three Variables and Applications

Example 5 Solving an Inconsistent System of Linear Equations

Solve the system. If there is not a unique solution, identify the system as either dependent or inconsistent.

\[
\begin{align*}
2x + 3y - 7z &= 4 \\
-4x - 6y + 14z &= 1 \\
5x + y - 3z &= 6
\end{align*}
\]

Solution:

We will eliminate the \(x\) variable from equations \(A\) and \(B\).

\[
\begin{align*}
A & \quad 2x + 3y - 7z = 4 \\
B & \quad -4x - 6y + 14z = 1
\end{align*}
\]

Multiply by 2:

\[
\begin{align*}
A & \quad 4x + 6y - 14z = 8 \\
B & \quad -4x - 6y + 14z = 1
\end{align*}
\]

\[
0 = 9
\]

(contradiction)

The result \(0 = 9\) is a contradiction, indicating that the system has no solution, \(\{\}\). The system is inconsistent.

Skill Practice

5. Solve the system. If the system does not have a unique solution, identify the system as dependent or inconsistent.

\[
\begin{align*}
x - 2y + z &= 5 \\
x - 3y + 2z &= -7 \\
-2x + 4y - 2z &= 6
\end{align*}
\]

Answer

5. No solution; inconsistent system

Section 3.6 Practice Exercises

Study Skills Exercise

1. Define the key terms.
   a. Linear equation in three variables
   b. Ordered triple

Review Exercises

2. Determine if the ordered pair \((-4, -7)\) is a solution to the system.

\[
\begin{align*}
-5x + 3y &= -1 \\
4x - 2y &= -2
\end{align*}
\]

For Exercises 3–4, solve the systems by using two methods: (a) the substitution method and (b) the addition method.

3. \(3x + y = 4\)
   
   \(4x + y = 5\)

4. \(2x - 5y = 3\)
   
   \(-4x + 10y = 3\)

5. Marge can ride her bike 24 mi in \(1\frac{1}{2}\) hr riding with the wind. Riding against the wind she can ride 24 mi in 2 hr. Find the speed at which Marge can ride in still air and the speed of the wind.

Concept 1: Solutions to Systems of Linear Equations in Three Variables

6. How many solutions are possible when solving a system of three equations with three variables?
Chapter 3  Systems of Linear Equations and Inequalities

7. Which of the following points are solutions to the system? \((2, 1, 7), (3, -10, -6), (4, 0, 2)\)
   \[
   \begin{align*}
   2x - y + z &= 10 \\
   4x + 2y - 3z &= 10 \\
   x - 3y + 2z &= 8
   \end{align*}
   \]

8. Which of the following points are solutions to the system? \((1, 1, 3), (0, 0, 4), (4, 2, 1)\)
   \[
   \begin{align*}
   -3x - 3y - 6z &= -24 \\
   -9x - 6y + 3z &= -45 \\
   9x + 3y - 9z &= 33
   \end{align*}
   \]

9. Which of the following points are solutions to the system? \((12, 2, -2), (4, 2, 1), (1, 1, 1)\)
   \[
   \begin{align*}
   -x - y - 4z &= -6 \\
   x - 3y + z &= -1 \\
   4x + y - z &= 4
   \end{align*}
   \]

10. Which of the following points are solutions to the system? \((0, 4, 3), (3, 6, 10), (3, 3, 1)\)
   \[
   \begin{align*}
   x + 2y - z &= 5 \\
   x - 3y + z &= -5 \\
   -2x + y - z &= -4
   \end{align*}
   \]

Concept 2: Solving Systems of Linear Equations in Three Variables

For Exercises 11–22, solve the system of equations. (See Example 1.)

11. \[
   \begin{align*}
   2x + y - 3z &= -12 \\
   3x - 2y - z &= 3 \\
   -x + 5y + 2z &= -3
   \end{align*}
   \]

12. \[
   \begin{align*}
   -3x - 2y + 4z &= -15 \\
   2x + 5y - 3z &= 3 \\
   4x - y + 7z &= 15
   \end{align*}
   \]

13. \[
   \begin{align*}
   x - 3y - 4z &= -7 \\
   5x + 2y + 2z &= -1 \\
   4x - y - 5z &= -6
   \end{align*}
   \]

14. \[
   \begin{align*}
   6x - 5y + z &= 7 \\
   5x + 3y + 2z &= 0 \\
   -2x + y - 3z &= 11
   \end{align*}
   \]

15. \[
   \begin{align*}
   4x + 2z &= 12 + 3y \\
   2y &= 3x + 3z - 5 \\
   y &= 2x + 7z + 8
   \end{align*}
   \]

16. \[
   \begin{align*}
   y &= 2x + z + 1 \\
   5x + 3z &= 16 - 3y
   \end{align*}
   \]

17. \[
   \begin{align*}
   x + y + z &= 6 \\
   -x + y - z &= -2 \\
   2x + 3y + z &= 11
   \end{align*}
   \]

18. \[
   \begin{align*}
   x - y - z &= -11 \\
   x + y - z &= 15 \\
   2x - y + z &= -9
   \end{align*}
   \]

19. \[
   \begin{align*}
   2x - 3y + 2z &= -1 \\
   x + 2y &= -4 \\
   x + z &= 1
   \end{align*}
   \]

20. \[
   \begin{align*}
   x + y + z &= 2 \\
   2x - z &= 5 \\
   3y + z &= 2
   \end{align*}
   \]

21. \[
   \begin{align*}
   4x + 9y &= 8 \\
   8x + 6z &= -1 \\
   6y + 6z &= -1
   \end{align*}
   \]

22. \[
   \begin{align*}
   3x + 2z &= 11 \\
   y - 7z &= 4 \\
   x - 6y &= 1
   \end{align*}
   \]

Concept 3: Applications of Linear Equations in Three Variables

23. A triangle has one angle that measures 5° more than twice the smallest angle, and the third angle measures 11° less than 3 times the measure of the smallest angle. Find the measures of the three angles. (See Example 2.)

24. The largest angle of a triangle measures 4° less than 5 times the measure of the smallest angle. The middle angle measures twice that of the smallest angle. Find the measures of the three angles.

25. The perimeter of a triangle is 55 cm. The measure of the shortest side is 8 cm less than the middle side. The measure of the longest side is 1 cm less than the sum of the other two sides. Find the lengths of the sides.

26. The perimeter of a triangle is 5 ft. The longest side of the triangle measures 20 in. more than the shortest side. The middle side is 3 times the measure of the shortest side. Find the lengths of the three sides in inches.

27. A movie theater charges $7 for adults, $5 for children under age 17, and $4 for seniors over age 60. For one showing of Batman the theater sold 222 tickets and took in $1383. If twice as many adult tickets were sold as the total of children and senior tickets, how many tickets of each kind were sold? (See Example 3.)

28. Baylor University in Waco, Texas, had twice as many students as Vanderbilt University in Nashville, Tennessee. Pace University in New York City had 2800 more students than Vanderbilt University. If the enrollment for all three schools totaled 27,200, find the enrollment for each school.
29. Goofie Golf has 18 holes that are par 3, par 4, or par 5. Most of the holes are par 4. In fact, there are 3 times as many par 4’s as par 3’s. There are 3 more par 5’s than par 3’s. How many of each type are there?

30. Combining peanuts, pecans, and cashews makes a party mixture of nuts. If the amount of peanuts equals the amount of pecans and cashews combined, and if there are twice as many cashews as pecans, how many ounces of each nut is used to make 48 oz of party mixture?

31. Souvenir hats, T-shirts, and jackets are sold at a rock concert. Three hats, two T-shirts, and one jacket cost $140. Two hats, two T-shirts, and two jackets cost $180. Find the prices of the individual items.

32. Annie and Maria traveled overseas for 7 days and stayed in three different hotels in three different cities: Stockholm, Sweden; Oslo, Norway; and Paris, France.

   The total bill for all seven nights (not including tax) was $1040. The total tax was $106. The nightly cost (excluding tax) to stay at the hotel in Paris was $80 more than the nightly cost (excluding tax) to stay in Oslo. Find the cost per night for each hotel excluding tax.

<table>
<thead>
<tr>
<th>City</th>
<th>Number of Nights</th>
<th>Cost/Night ($)</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris, France</td>
<td>1</td>
<td>x</td>
<td>8%</td>
</tr>
<tr>
<td>Stockholm, Sweden</td>
<td>4</td>
<td>y</td>
<td>11%</td>
</tr>
<tr>
<td>Oslo, Norway</td>
<td>2</td>
<td>z</td>
<td>10%</td>
</tr>
</tbody>
</table>

Concept 4: Solving Dependent and Inconsistent Systems (Mixed Exercises)

For Exercises 33–44, solve the system. If there is not a unique solution, label the system as either dependent or inconsistent. (See Examples 1, 4, and 5.)

33. \[2x + y + 3z = 2\]
   \[-x - y + 2z = -4\]
   \[2x + 2y - 4z = 8\]

34. \[x + y = z\]
   \[2x + 4y - 2z = 6\]
   \[3x + 6y - 3z = 9\]

35. \[6x - 2y + 2z = 2\]
   \[4x + 8y - 2z = 5\]
   \[-2x - 4y + z = -2\]

36. \[3x + 2y + z = 3\]
   \[x - 3y + z = 4\]
   \[-6x - 4y - 2z = 1\]

37. \[\frac{1}{2}x + \frac{3}{2}y = \frac{5}{2}\]
   \[\frac{1}{5}x - \frac{1}{2}z = -\frac{3}{10}\]
   \[\frac{1}{2}y - \frac{1}{2}z = \frac{4}{5}\]

38. \[\frac{1}{3}x + \frac{1}{4}y + \frac{1}{3}z = \frac{1}{3}\]
   \[\frac{1}{3}x + \frac{1}{3}y + \frac{1}{3}z = \frac{3}{8}\]
   \[x - y - \frac{3}{4}z = \frac{1}{2}\]

39. \[-3x + y - z = 8\]
   \[-4x + 2y + 3z = -3\]
   \[2x + 3y - 2z = -1\]

40. \[2x + 3y + 3z = 15\]
   \[3x - 6y - 6z = -23\]
   \[-9x - 3y + 6z = 8\]

41. \[2x + y = 3(z - 1)\]
   \[3x - 2(y - z) = 1\]
   \[2(2x - 3z) = -6 - 2y\]

42. \[2x + y = -3\]
   \[2y + 16z = -10\]
   \[-7x - 3y + 4z = 8\]

43. \[-0.1y + 0.2z = 0.2\]
   \[0.1x + 0.1y + 0.1z = 0.2\]
   \[-0.1x + 0.3z = 0.2\]

44. \[-0.4x - 0.3y = 0\]
   \[0.3y + 0.1z = -0.1\]
   \[0.4x - 0.1z = 1.2\]

Expanding Your Skills

The systems in Exercises 45–48 are called homogeneous systems because each system has \((0, 0, 0)\) as a solution. However, if a system is dependent, it will have infinitely many more solutions. For each system determine whether \((0, 0, 0)\) is the only solution or if the system is dependent.

45. \[2x - 4y + 8z = 0\]
   \[-x - 3y + z = 0\]
   \[x - 2y + 5z = 0\]

46. \[2x - 4y + z = 0\]
   \[x - 3y - z = 0\]
   \[3x - y + 2z = 0\]

47. \[4x - 2y - 3z = 0\]
   \[-8x - y + z = 0\]
   \[2x - y - \frac{3}{2}z = 0\]

48. \[5x + y = 0\]
   \[4y - z = 0\]
   \[5x + 5y - z = 0\]
Section 3.7

Solving Systems of Linear Equations by Using Matrices

1. Introduction to Matrices

In Sections 3.2, 3.3, and 3.6, we solved systems of linear equations by using the substitution method and the addition method. We now present a third method called the Gauss-Jordan method that uses matrices to solve a linear system.

A matrix is a rectangular array of numbers (the plural of matrix is matrices). The rows of a matrix are read horizontally, and the columns of a matrix are read vertically. Every number or entry within a matrix is called an element of the matrix.

The order of a matrix is determined by the number of rows and number of columns. A matrix with \( m \) rows and \( n \) columns is an \( m \times n \) (read as “\( m \) by \( n \)”)

matrix. Notice that with the order of a matrix, the number of rows is given first, followed by the number of columns.

Example 1  Determining the Order of a Matrix

Determine the order of each matrix.

\[
A = \begin{bmatrix} 2 & -4 & 1 \\ 5 & \pi & \sqrt{7} \end{bmatrix}  \quad B = \begin{bmatrix} 1.9 \\ 0 \\ 7.2 \\ -6.1 \end{bmatrix}  \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}  \quad D = \begin{bmatrix} a & b & c \end{bmatrix}
\]

Solution:

a. This matrix has two rows and three columns. Therefore, it is a \( 2 \times 3 \) matrix.

b. This matrix has four rows and one column. Therefore, it is a \( 4 \times 1 \) matrix. A matrix with one column is called a column matrix.

c. This matrix has three rows and three columns. Therefore, it is a \( 3 \times 3 \) matrix. A matrix with the same number of rows and columns is called a square matrix.

d. This matrix has one row and three columns. Therefore, it is a \( 1 \times 3 \) matrix. A matrix with one row is called a row matrix.

Skill Practice  Determine the order of the matrix.

1. \( \begin{bmatrix} -5 & 2 \\ 1 & 3 \\ 8 & 9 \end{bmatrix} \)  2. \( \begin{bmatrix} 4 & -8 \end{bmatrix} \)  3. \( \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} \)  4. \( \begin{bmatrix} 2 & -0.5 \\ -1 & 6 \end{bmatrix} \)

A matrix can be used to represent a system of linear equations written in standard form. To do so, we extract the coefficients of the variable terms and the constants within the equation. For example, consider the system

\[
\begin{align*}
2x - y &= 5 \\
x + 2y &= -5
\end{align*}
\]

The matrix \( A \) is called the coefficient matrix.

\[
A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}
\]

Answers

1. \( 3 \times 2 \)  2. \( 1 \times 2 \)  3. \( 3 \times 1 \)  4. \( 2 \times 2 \)
If we extract both the coefficients and the constants from the equations, we can construct the augmented matrix of the system:

\[
\begin{bmatrix}
2 & -1 & 5 \\
1 & 2 & -5
\end{bmatrix}
\]

A vertical bar is inserted into an augmented matrix to designate the position of the equal signs.

**Example 2** Writing an Augmented Matrix for a System of Linear Equations

Write the augmented matrix for each linear system.

a. \(-3x - 4y = 3\)  
b. \(2x - 3z = 14\)  
\[\begin{align*}
2x + 4y &= 2 \\
x + y &= 4
\end{align*}\]

**Solution:**

a. \[
\begin{bmatrix}
-3 & -4 & 3 \\
2 & 4 & 2
\end{bmatrix}
\]
b. \[
\begin{bmatrix}
2 & 0 & -3 & 14 \\
0 & 2 & 1 & 2 \\
1 & 1 & 0 & 4
\end{bmatrix}
\]

**Skill Practice** Write the augmented matrix for each system.

5. \(-x + y = 4\)  
6. \(2x - y + z = 14\)  
\[\begin{align*}
2x - y &= 1 \\
-3x + 4y &= 8 \\
x - y + 5z &= 0
\end{align*}\]

**Example 3** Writing a Linear System from an Augmented Matrix

Write a system of linear equations represented by each augmented matrix.

a. \[
\begin{bmatrix}
2 & -5 & -8 \\
4 & 1 & 6
\end{bmatrix}
\]
b. \[
\begin{bmatrix}
2 & -1 & 3 & 14 \\
1 & 1 & -2 & -5 \\
3 & 1 & -1 & 2
\end{bmatrix}
\]
c. \[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

**Solution:**

a. \(2x - 5y = -8\)  
\(4x + y = 6\)  
\(3x + y - z = 2\)

b. \(2x - y + 3z = 14\)  
\(x + y - 2z = -5\)  
\(3x + y - z = 2\)

c. \(x + 0y + 0z = 4\)  
\(0x + y + 0z = -1\)  
\(0x + 0y + z = 0\)

**Skill Practice** Write a system of linear equations represented by each augmented matrix.

7. \[
\begin{bmatrix}
2 & 3 & 5 \\
-1 & 8 & 1
\end{bmatrix}
\]
8. \[
\begin{bmatrix}
-3 & 2 & 1 & 4 \\
14 & 1 & 0 & 20
\end{bmatrix}
\]
9. \[
\begin{bmatrix}
1 & 0 & 0 & -8 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 15
\end{bmatrix}
\]

**Answers**

5. \[
\begin{bmatrix}
-1 & 1 & 4 \\
2 & -1 & 1
\end{bmatrix}
\]
6. \[
\begin{bmatrix}
-3 & 4 & 0 & 8 \\
1 & -1 & 5 & 0
\end{bmatrix}
\]
7. \(2x + 3y = 5\)  
\(-x + 8y = 1\)

8. \(-3x + 2y + z = 4\)  
\(14x + y = 20\)  
\(-8x + 3y + 5z = 6\)

9. \(x = -8, y = 2, z = 15\)
Chapter 3  Systems of Linear Equations and Inequalities

2. Solving Systems of Linear Equations by Using the Gauss-Jordan Method

We know that interchanging two equations results in an equivalent system of linear equations. Interchanging two rows in an augmented matrix results in an equivalent augmented matrix. Similarly, because each row in an augmented matrix represents a linear equation, we can perform the following elementary row operations that result in an equivalent augmented matrix.

**PROPERTY  Elementary Row Operations**

The following *elementary row operations* performed on an augmented matrix produce an equivalent augmented matrix:

- Interchange two rows.
- Multiply every element in a row by a nonzero real number.
- Add a multiple of one row to another row.

When we are solving a system of linear equations by any method, the goal is to write a series of simpler but equivalent systems of equations until the solution is obvious. The *Gauss-Jordan method* uses a series of elementary row operations performed on the augmented matrix to produce a simpler augmented matrix. In particular, we want to produce an augmented matrix that has 1’s along the diagonal of the matrix of coefficients and 0’s for the remaining entries in the matrix of coefficients. A matrix written in this way is said to be written in *reduced row echelon form*. For example, the augmented matrix from Example 3(c) is written in reduced row echelon form.

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

The solution to the corresponding system of equations is easily recognized as \(x = 4, y = -1, \text{ and } z = 0\).

Similarly, matrix \(B\) represents a solution of \(x = a\) and \(y = b\).

\[
B = \begin{bmatrix}
1 & 0 & a \\
0 & 1 & b \\
\end{bmatrix}
\]

**Example 4  Solving a System by Using the Gauss-Jordan Method**

Solve by using the Gauss-Jordan method.

\[
\begin{align*}
2x - y &= 5 \\
x + 2y &= -5 \\
\end{align*}
\]
Section 3.7 Solving Systems of Linear Equations by Using Matrices

### Solution:

\[
\begin{bmatrix}
2 & -1 & 5 \\
1 & 2 & -5
\end{bmatrix}
\]

Set up the augmented matrix.

\[
R_1 \leftrightarrow R_2
\]

\[
\begin{bmatrix}
1 & 2 & -5 \\
2 & -1 & 5
\end{bmatrix}
\]

Switch row 1 and row 2 to get a 1 in the upper left position.

\[
-2R_1 + R_2 \Rightarrow R_2
\]

\[
\begin{bmatrix}
1 & 2 & -5 \\
0 & -5 & 15
\end{bmatrix}
\]

Multiply row 1 by \(-2\) and add the result to row 2. This produces an entry of 0 below the upper left position.

\[
-\frac{1}{5}R_2 \Rightarrow R_2
\]

\[
\begin{bmatrix}
1 & 2 & -5 \\
0 & 1 & -3
\end{bmatrix}
\]

Multiply row 2 by \(-\frac{1}{5}\) to produce a 1 along the diagonal in the second row.

\[
-2R_2 + R_1 \Rightarrow R_1
\]

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -3
\end{bmatrix}
\]

Multiply row 2 by \(-2\) and add the result to row 1. This produces a 0 in the first row, second column.

\[
C = \begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & -3
\end{bmatrix}
\]

The matrix \(C\) is in reduced row echelon form. From the augmented matrix, we have \(x = 1\) and \(y = -3\). The solution set is \((1, -3)\).

### Skill Practice

10. Solve by using the Gauss-Jordan method.

\[
\begin{align*}
x - 2y &= -21 \\
2x + y &= -2
\end{align*}
\]

The order in which we manipulate the elements of an augmented matrix to produce reduced row echelon form was demonstrated in Example 4. In general, the order is as follows.

- First produce a 1 in the first row, first column. Then use the first row to obtain 0's in the first column below this element.
- Next, if possible, produce a 1 in the second row, second column. Use the second row to obtain 0's above and below this element.
- Next, if possible, produce a 1 in the third row, third column. Use the third row to obtain 0's above and below this element.
- The process continues until reduced row echelon form is obtained.

### Answer

10. \((-5, 8)\)
Example 5  Solving a System by Using the Gauss-Jordan Method

Solve by using the Gauss-Jordan method.

\[
\begin{align*}
    x &= -y + 5 \\
    -2x + 2z &= y - 10 \\
    3x + 6y + 7z &= 14
\end{align*}
\]

Solution:

First write each equation in the system in standard form.

\[
\begin{align*}
    x &= -y + 5 \\
    -2x + 2z &= y - 10 \\
    3x + 6y + 7z &= 14
\end{align*}
\]

Set up the augmented matrix.

\[
\begin{bmatrix}
    1 & 1 & 0 & | & 5 \\
    -2 & -1 & 2 & | & -10 \\
    3 & 6 & 7 & | & 14
\end{bmatrix}
\]

Multiply row 1 by 2 and add the result to row 2. Multiply row 1 by \(\frac{1}{2}\) and add the result to row 3.

Multiply row 2 by \(-1\) and add the result to row 1. Multiply row 2 by \(-3\) and add the result to row 3.

Multiply row 3 by 2 and add the result to row 1. Multiply row 3 by \(-\frac{1}{2}\) and add the result to row 2.

From the reduced row echelon form of the matrix, we have \(x = 3\), \(y = 2\), and \(z = -1\). The solution set is \((3, 2, -1)\).

Skill Practice  Solve by using the Gauss-Jordan method.

11. \(x + y + z = 2\)
    \(x - y + z = 4\)
    \(x + 4y + 2z = 1\)

Answer  

11. \((1, -1, 2)\)
It is particularly easy to recognize a dependent or inconsistent system of equations from the reduced row echelon form of an augmented matrix. This is demonstrated in Examples 6 and 7.

**Example 6** Solving a Dependent System by Using the Gauss-Jordan Method

Solve by using the Gauss-Jordan method.

\[
\begin{align*}
  x & - 3y = 4 \\
  \frac{1}{2}x - \frac{3}{2}y = 2
\end{align*}
\]

**Solution:**

\[
\begin{bmatrix}
1 & -3 & 4 \\
\frac{1}{2} & -\frac{3}{2} & 2
\end{bmatrix}
\]

Set up the augmented matrix.

\[
\begin{align*}
-\frac{1}{2}R_1 + R_2 & \Rightarrow R_2 \\
\begin{bmatrix}
1 & -3 & 4 \\
0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

Multiply row 1 by \(-\frac{1}{2}\) and add the result to row 2.

The second row of the augmented matrix represents the equation 0 = 0. The system is dependent. The solution set is \(\{(x, y) \mid x - 3y = 4\}\).

**Skill Practice** Solve by using the Gauss-Jordan method.

12. \(4x - 6y = 16\) \(6x - 9y = 24\)

**Example 7** Solving an Inconsistent System by Using the Gauss-Jordan Method

Solve by using the Gauss-Jordan method.

\[
\begin{align*}
  x & + 3y = 2 \\
  -3x & - 9y = 1
\end{align*}
\]

**Solution:**

\[
\begin{bmatrix}
1 & 3 & 2 \\
-3 & -9 & 1
\end{bmatrix}
\]

Set up the augmented matrix.

\[
\begin{align*}
3R_1 + R_2 & \Rightarrow R_2 \\
\begin{bmatrix}
1 & 3 & 2 \\
0 & 0 & 7
\end{bmatrix}
\end{align*}
\]

Multiply row 1 by 3 and add the result to row 2.

The second row of the augmented matrix represents the contradiction 0 = 7. The system is inconsistent. There is no solution, \(\{\}\).

**Skill Practice** Solve by using the Gauss-Jordan method.

13. \(6x + 10y = 1\) \(15x + 25y = 3\)

**Answers**

12. Infinitely many solutions; \(\{(x, y) \mid 4x - 6y = 16\}\); dependent system
13. No solution; \(\{\}\); inconsistent system
Chapter 3  Systems of Linear Equations and Inequalities

**Calculator Connections**

**Topic: Entering a Matrix into a Calculator**

Many graphing calculators have a matrix editor in which the user defines the order of the matrix and then enters the elements of the matrix. For example, the $2 \times 3$ matrix

$$D = \begin{bmatrix} 2 & -3 & -13 \\ 3 & 1 & 8 \end{bmatrix}$$

is entered as shown.

Once an augmented matrix has been entered into a graphing calculator, a `rref` function can be used to transform the matrix into reduced row echelon form.

---

**Section 3.7  Practice Exercises**

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**Study Skills Exercise**

1. Define the key terms.
   
   a. Matrix   
   b. Order of a matrix   
   c. Column matrix   
   d. Square matrix
   
   e. Row matrix   
   f. Coefficient matrix   
   g. Augmented matrix   
   h. Reduced row echelon form

**Review Exercises**

2. How much 50% acid solution should be mixed with pure acid to obtain 20 L of a mixture that is 70% acid?

For Exercises 3–5, solve the system by using any method.

3. $\begin{align*} x - 6y &= 9 \\
    x + 2y &= 13 \end{align*}$

4. $\begin{align*} x + y - z &= 8 \\
    x - 2y + z &= 3 \end{align*}$

5. $\begin{align*} 2x - y + z &= -4 \\
    -x + y + 3z &= -7 \end{align*}$
Section 3.7 Solving Systems of Linear Equations by Using Matrices

Concept 1: Introduction to Matrices
For Exercises 6–14, (a) determine the order of each matrix and (b) determine if the matrix is a row matrix, a column matrix, a square matrix, or none of these. (See Example 1.)

6. \[
\begin{bmatrix}
4 \\
5 \\
-3 \\
0
\end{bmatrix}
\]
7. \[
\begin{bmatrix}
5 \\
-1 \\
2
\end{bmatrix}
\]
8. \[
\begin{bmatrix}
-9 & 4 & 3 \\
-1 & -8 & 4 \\
5 & 8 & 7
\end{bmatrix}
\]
9. \[
\begin{bmatrix}
3 & -9 \\
-1 & -3
\end{bmatrix}
\]
10. [4 -7]
11. [0 -8 11 5]
12. \[
\begin{bmatrix}
5 & -8.1 & 4.2 & 0 \\
4.3 & -9 & 18 & 3
\end{bmatrix}
\]
13. \[
\begin{bmatrix}
\frac{1}{2} & \frac{3}{2} \\
-2 & 1 & \frac{6}{3}
\end{bmatrix}
\]
14. \[
\begin{bmatrix}
5 & 1 \\
-1 & 2 \\
0 & 7
\end{bmatrix}
\]

For Exercises 15–18, set up the augmented matrix. (See Example 2.)
15. \[x - 2y = -1
\]
\[2x + y = -7 \]
16. \[x - 3y = 3
\]
\[2x - 5y = 4 \]
17. \[x - 2y = 5 - z
\]
\[2x + 6y + 3z = -2
\]
\[3x - y - 2z = 1 \]
18. \[5x - 17 = -2z
\]
\[8x + 6z = 26 + y
\]
\[8x + 3y - 12z = 24 \]

For Exercises 19–22, write a system of linear equations represented by the augmented matrix. (See Example 3.)
19. \[
\begin{bmatrix}
4 & 3 & 6 \\
12 & 5 & -6
\end{bmatrix}
\]
20. \[
\begin{bmatrix}
-2 & 5 & -15 \\
-7 & 15 & -45
\end{bmatrix}
\]
21. \[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 7
\end{bmatrix}
\]
22. \[
\begin{bmatrix}
1 & 0 & 0 & 0.5 \\
0 & 1 & 0 & 6.1 \\
0 & 0 & 1 & 3.9
\end{bmatrix}
\]

Concept 2: Solving Systems of Linear Equations by Using the Gauss-Jordan Method
23. Given the matrix \(E\)
\[
E = \begin{bmatrix}
3 & -2 & 8 \\
9 & -1 & 7
\end{bmatrix}
\]
a. What is the element in the second row and third column?
b. What is the element in the first row and second column?
24. Given the matrix \(F\)
\[
F = \begin{bmatrix}
1 & 8 & 0 \\
12 & -13 & -2
\end{bmatrix}
\]
a. What is the element in the second row and second column?
b. What is the element in the first row and third column?
25. Given the matrix \(Z\)
\[
Z = \begin{bmatrix}
2 & 1 & 11 \\
2 & -1 & 1
\end{bmatrix}
\]
write the matrix obtained by multiplying the elements in the first row by \(\frac{1}{2}\).
26. Given the matrix \(J\)
\[
J = \begin{bmatrix}
1 & 1 & 7 \\
0 & 3 & -6
\end{bmatrix}
\]
write the matrix obtained by multiplying the elements in the second row by \(\frac{1}{3}\).
27. Given the matrix \(K\)
\[
K = \begin{bmatrix}
5 & 2 & 1 \\
1 & -4 & 3
\end{bmatrix}
\]
write the matrix obtained by interchanging rows 1 and 2.
28. Given the matrix \(L\)
\[
L = \begin{bmatrix}
9 & 6 & 13 \\
-7 & 2 & 19
\end{bmatrix}
\]
write the matrix obtained by interchanging rows 1 and 2.
29. Given the matrix \[
M = \begin{bmatrix}
  1 & 5 \\
-3 & 4 \\
\end{bmatrix}
\]
write the matrix obtained by multiplying the first row by 3 and adding the result to row 2.

30. Given the matrix \[
N = \begin{bmatrix}
  1 & 3 & -5 \\
-2 & 2 & 12 \\
\end{bmatrix}
\]
write the matrix obtained by multiplying the first row by 2 and adding the result to row 2.

31. Given the matrix \[
R = \begin{bmatrix}
  1 & 3 & 0 & -1 \\
4 & 1 & -5 & 6 \\
-2 & 0 & -3 & 10 \\
\end{bmatrix}
\]
   a. Write the matrix obtained by multiplying the first row by \(-4\) and adding the result to row 2.
   b. Using the matrix obtained from part (a), write the matrix obtained by multiplying the first row by 2 and adding the result to row 3.

32. Given the matrix \[
S = \begin{bmatrix}
  1 & 2 & 0 & 10 \\
5 & 1 & -4 & 3 \\
-3 & 4 & 5 & 2 \\
\end{bmatrix}
\]
   a. Write the matrix obtained by multiplying the first row by \(-5\) and adding the result to row 2.
   b. Using the matrix obtained from part (a), write the matrix obtained by multiplying the first row by 3 and adding the result to row 3.

For Exercises 33–48, solve the systems by using the Gauss-Jordan method. (See Examples 4–7.)

33. \[
\begin{align*}
x - 2y &= -1 \\
2x + y &= -7 \\
\end{align*}
\]
34. \[
\begin{align*}
x - 3y &= 3 \\
2x - 5y &= 4 \\
\end{align*}
\]
35. \[
\begin{align*}
x + 3y &= 6 \\
-4x - 9y &= 3 \\
\end{align*}
\]
36. \[
\begin{align*}
x - 3y &= -2 \\
x + 2y &= 13 \\
\end{align*}
\]
37. \[
\begin{align*}
x + 3y &= 3 \\
4x + 12y &= 12 \\
\end{align*}
\]
38. \[
\begin{align*}
2x + 5y &= 1 \\
-4x - 10y &= -2 \\
\end{align*}
\]
39. \[
\begin{align*}
x - y &= 4 \\
2x + y &= 5 \\
\end{align*}
\]
40. \[
\begin{align*}
x - y &= 0 \\
x + y &= 3 \\
\end{align*}
\]
41. \[
\begin{align*}
x + 3y &= -1 \\
-3x - 6y &= 12 \\
\end{align*}
\]
42. \[
\begin{align*}
x + y &= 4 \\
2x - 4y &= -4 \\
\end{align*}
\]
43. \[
\begin{align*}
3x + y &= -4 \\
-6x - 2y &= 3 \\
\end{align*}
\]
44. \[
\begin{align*}
2x + y &= 4 \\
6x + 3y &= -1 \\
\end{align*}
\]
45. \[
\begin{align*}
x + y + z &= 6 \\
x - y + z &= 2 \\
x + y - z &= 0 \\
\end{align*}
\]
46. \[
\begin{align*}
2x - 3y - 2z &= 11 \\
x + 3y + 8z &= 1 \\
3x - y + 14z &= -2 \\
\end{align*}
\]
47. \[
\begin{align*}
x - 2y &= 5 - z \\
x - 2y &= 5 - z \\
\end{align*}
\]
48. \[
\begin{align*}
5x &= 10z + 15 \\
x - y + 6z &= 23 \\
x + 3y - 12z &= 13 \\
\end{align*}
\]

For Exercises 49–52, use the augmented matrices \(A, B, C,\) and \(D\) to answer true or false.

\[
A = \begin{bmatrix}
  6 & -4 & 2 \\
 5 & -2 & 7 \\
\end{bmatrix} \quad B = \begin{bmatrix}
  5 & -2 & 7 \\
 6 & -4 & 2 \\
\end{bmatrix} \quad C = \begin{bmatrix}
  1 & -\frac{1}{2} & \frac{1}{2} \\
 5 & -2 & 7 \\
\end{bmatrix} \quad D = \begin{bmatrix}
  5 & -2 & 7 \\
-12 & 8 & -4 \\
\end{bmatrix}
\]

49. The matrix \(A\) is a \(2 \times 3\) matrix.

50. Matrix \(B\) is equivalent to matrix \(A\).

51. Matrix \(A\) is equivalent to matrix \(C\).

52. Matrix \(B\) is equivalent to matrix \(D\).

53. What does the notation \(R_2 \leftrightarrow R_3\) mean when one is performing the Gauss-Jordan method?

54. What does the notation \(2R_3 \Rightarrow R_1\) mean when one is performing the Gauss-Jordan method?

55. What does the notation \(-3R_1 + R_2 \Rightarrow R_2\) mean when one is performing the Gauss-Jordan method?

56. What does the notation \(4R_2 + R_3 \Rightarrow R_3\) mean when one is performing the Gauss-Jordan method?
Graphing Calculator Exercises

For Exercises 57–62, use the matrix features on a graphing calculator to express each augmented matrix in reduced row echelon form. Compare your results to the solution you obtained in the indicated exercise.

57. \[
\begin{bmatrix}
1 & -2 & -1 \\
2 & 1 & -7 \\
\end{bmatrix}
\]

Compare with Exercise 33.

58. \[
\begin{bmatrix}
1 & -3 & 3 \\
2 & -5 & 4 \\
\end{bmatrix}
\]

Compare with Exercise 34.

59. \[
\begin{bmatrix}
1 & 3 & 6 \\
-4 & -9 & 3 \\
\end{bmatrix}
\]

Compare with Exercise 35.

60. \[
\begin{bmatrix}
2 & -3 & -2 \\
1 & 2 & 13 \\
\end{bmatrix}
\]

Compare with Exercise 36.

61. \[
\begin{bmatrix}
1 & 1 & 1 & 6 \\
1 & -1 & 1 & 2 \\
1 & 1 & -1 & 0 \\
\end{bmatrix}
\]

Compare with Exercise 45.

62. \[
\begin{bmatrix}
2 & -3 & -2 & 11 \\
1 & 3 & 8 & 1 \\
3 & -1 & 14 & -2 \\
\end{bmatrix}
\]

Compare with Exercise 46.

Creating a Quadratic Model of the Form \( y = at^2 + bt + c \)

Estimated time: 20 minutes

Group Size: 3

Natalie Dalton was a player on the Daytona State College women’s fast-pitch softball team. She threw a ball 120 ft from right field to make a play at third base. A photographer used strobe photography to follow the path of the ball. The height of the ball (in ft) at 0.2-sec intervals was recorded in the table.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>5</td>
<td>11</td>
<td>16</td>
<td>19</td>
<td>21</td>
<td>22</td>
<td>21</td>
<td>19</td>
<td>16</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

1. Plot the points \((t, y)\) from the data.

2. Select three data points and label them \((t_1, y_1), (t_2, y_2),\) and \((t_3, y_3).\)

   \((t_1, y_1) = \quad \) 

   \((t_2, y_2) = \quad \) 

   \((t_3, y_3) = \quad \) 

3. Substitute the values of \(t_i\) and \(y_i\) for \(t\) and \(y\) in the quadratic model \( y = at^2 + bt + c.\) The equation you are left with should only have the variables \(a, b,\) and \(c.\) Call this equation 1. Then repeat this step two more times, using the points \((t_2, y_2)\) and \((t_3, y_3).\) You should then be left with three equations involving \(a, b,\) and \(c\) only.

   Equation 1: \(\quad \) 

   Equation 2: \(\quad \) 

   Equation 3: \(\quad \)
4. Solve the system of equations from step 3 for the variables \(a\), \(b\), and \(c\).

5. Replace the values of \(a\), \(b\), and \(c\) into the quadratic function \(y = ar^2 + bt + c\).

6. Use the function found in step 5 to approximate the height of the ball after \(\frac{1}{2}\) sec.

7. Use the function found in step 5 to approximate the height of the ball after 1.4 seconds. How well does this value match the observed value of 19 ft?

---

**Chapter 3 Summary**

**Section 3.1 Solving Systems of Linear Equations by the Graphing Method**

**Key Concepts**

A **system of linear equations** in two variables can be solved by graphing.

A **solution to a system of linear equations** is an ordered pair that satisfies each equation in the system. Graphically, this represents a point of intersection of the lines.

There may be one solution, infinitely many solutions, or no solution.

![Graph showing one solution, infinitely many solutions, and no solution]

A system of equations is **consistent** if there is at least one solution. A system is **inconsistent** if there is no solution.

A linear system in \(x\) and \(y\) is **dependent** if two equations represent the same line. The solution set is the set of all points on the line.

If two linear equations represent different lines, then the system of equations is **independent**.

**Examples**

**Example 1**

Solve by graphing.  
\[x + y = 3\]
\[2x - y = 0\]

Write each equation in slope-intercept form \((y = mx + b)\) to graph the lines.

\[y = -x + 3\]
\[y = 2x\]

The solution is the point of intersection, \((1, 2)\).
The solution set is \{(1, 2)\}. 
Section 3.2  Solving Systems of Linear Equations by the Substitution Method

Key Concepts

**Substitution Method**
1. Isolate one of the variables.
2. Substitute the quantity found in step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the value from step 3 back into the equation from step 1 to solve for the remaining variable.
5. Check the ordered pair in both equations, and write the answer as an ordered pair within set notation.

A system is consistent if there is at least one solution. A system is inconsistent if there is no solution. An inconsistent system is detected by a contradiction (such as $0 = 5$).

A system is independent if the two equations represent different lines. A system is dependent if the two equations represent the same line. This produces infinitely many solutions. A dependent system is detected by an identity (such as $0 = 0$).

Examples

**Example 1**

\[
\begin{align*}
2y &= -6x + 14 \\
2x + y &= 5 \\
\end{align*}
\]

Isolate a variable: $y = -2x + 5$

Substitute:

\[
\begin{align*}
2(-2x + 5) &= -6x + 14 \\
-4x + 10 &= -6x + 14 \\
2x &= 4 \\
x &= 2 \\
y &= -2(2) + 5 \\
y &= 1
\end{align*}
\]

Now solve for $y$.

The ordered pair $(2, 1)$ checks in both equations.

The solution set is $\{(2, 1)\}$.

**Example 2**

\[
\begin{align*}
y &= -2x + 3 \\n-4x - 2y &= 1 \\
\end{align*}
\]

Contradiction. The system is inconsistent. There is no solution, $\emptyset$.

**Example 3**

\[
\begin{align*}
x &= -3y + 1 \\
2x + 6y &= 2 \\
\end{align*}
\]

Identity. The system is dependent. There are infinitely many solutions.

\[(x, y) | x = -3y + 1\]
Section 3.3  Solving Systems of Linear Equations by the Addition Method

Key Concepts

Addition Method
1. Write both equations in standard form $Ax + By = C$.
2. Clear fractions or decimals (optional).
3. Multiply one or both equations by nonzero constants to create opposite coefficients for one of the variables.
4. Add the equations from step 3 to eliminate one variable.
5. Solve for the remaining variable.
6. Substitute the known value from step 5 back into one of the original equations to solve for the other variable.
7. Check the ordered pair in both equations and write the solution set.

Examples

Example 1

\[
\begin{align*}
3x - 4y &= 18 \\
-5x - 3y &= -1
\end{align*}
\]

\[
\begin{align*}
9x - 12y &= 54 \\
20x + 12y &= 4
\end{align*}
\]

\[
\begin{align*}
29x &= 58 \\
x &= 2
\end{align*}
\]

\[
\begin{align*}
3(2) - 4y &= 18 \\
6 - 4y &= 18 \\
-4y &= 12 \\
y &= -3
\end{align*}
\]

The ordered pair $(2, -3)$ checks in both equations.

The solution set is $\{(2, -3)\}$. 
Section 3.4 Applications of Systems of Linear Equations in Two Variables

Key Concepts
Solve application problems by using systems of linear equations in two variables.

- Cost applications
- Mixture applications
- Principal and interest applications
- Uniform motion applications
- Geometry applications

Steps to Solve Applications:
1. Label two variables.
2. Construct two equations in words.
3. Write two equations.
4. Solve the system.
5. Write the answer.

Examples

Example 1
Mercedes invested $1500 more in a certificate of deposit that pays 6.5% simple interest than she did in a savings account that pays 4% simple interest. If her total interest at the end of 1 yr is $622.50, find the amount she invested in the 6.5% account.

Let \( x \) represent the amount of money invested at 6.5%.
Let \( y \) represent the amount of money invested at 4%.

\[
\begin{align*}
\text{Interest earned from 6.5\% account} & = 0.065x \\
\text{Interest earned from 4\% account} & = 0.04y \\
\text{Total interest} & = 622.50
\end{align*}
\]

\[
\begin{align*}
\text{Amount invested at 6.5\%} & = \text{Amount invested at 4\%} + 1500 \\
0.065x + 0.04y & = 622.50
\end{align*}
\]

Using substitution gives

\[
\begin{align*}
0.065(y + 1500) + 0.04y & = 622.50 \\
0.065y + 97.5 + 0.04y & = 622.50 \\
0.105y & = 525 \\
y & = 5000
\end{align*}
\]

\[
x = (5000) + 1500 = 6500
\]

Mercedes invested $6500 at 6.5%.
Chapter 3  Systems of Linear Equations and Inequalities

Section 3.5  Linear Inequalities and Systems of Linear Inequalities in Two Variables

Key Concepts
A linear inequality in two variables is an inequality of the form $ax + by < c$, $ax + by > c$, $ax + by \leq c$, or $ax + by \geq c$.

Graphing a Linear Inequality in Two Variables
1. Solve for $y$, if possible.
2. Graph the related equation. Draw a dashed line if the inequality is strict, < or >. Otherwise, draw a solid line.
3. Shade above or below the line according to the following convention.
   - Shade above the line if the inequality is of the form $y > ax + b$ or $y \geq ax + b$.
   - Shade below the line if the inequality is of the form $y < ax + b$ or $y \leq ax + b$.

You can use test points to check that you have shaded the correct region. Select an ordered pair from the proposed solution set and substitute the values of $x$ and $y$ in the original inequality. If the test point produces a true statement, then you have shaded the correct region.

The union or intersection of two or more linear inequalities is the union or intersection of the solution sets.

Examples

Example 1
Graph the solution to the inequality $2x - y < 4$.
Solve for $y$: $2x - y < 4$
$-y < -2x + 4$
$y > 2x - 4$

Graph the related equation, $y = 2x - 4$, with a dashed line.
Shade above the line.

Example 2
Graph the solution set.
$x < 0$ and $y > 2$

Example 3
Graph the solution set.
$x \leq 0$ or $y \geq 2$
Summary

Section 3.6 Systems of Linear Equations in Three Variables and Applications

Key Concepts

A linear equation in three variables can be written in the form $Ax + By + Cz = D$, where $A$, $B$, and $C$ are not all zero. The graph of a linear equation in three variables is a plane in space.

A solution to a system of linear equations in three variables is an ordered triple that satisfies each equation. Graphically, a solution is a point of intersection among three planes.

A system of linear equations in three variables may have one unique solution, infinitely many solutions (dependent system), or no solution (inconsistent system).

Examples

Example 1

Substitute into either equation or .

Substitute and into equation , , or .

The solution set is $\{2, 1, 0\}$. 

Multiply by $-1$.

Substitute $z = 1$ into either equation $D$ or $E$.

$D \quad 4x + y = 9 \\
E \quad 4x + 7y = 15$

$4x + (1) = 9 \\
4x = 8 \\
x = 2$

Substitute $x = 2$ and $y = 1$ into equation $A$, $B$, or $C$.

$A \quad (2) + (1) - z = 4 \\
z = 0$

The solution set is $\{2, 1, 0\}$. 

Section 3.7 Solving Systems of Linear Equations by Using Matrices

**Key Concepts**

A **matrix** is a rectangular array of numbers displayed in rows and columns. Every number or entry within a matrix is called an element of the matrix.

The **order of a matrix** is determined by the number of rows and number of columns. A matrix with \( m \) rows and \( n \) columns is an \( m \times n \) matrix.

A system of equations written in standard form can be represented by an **augmented matrix** consisting of the coefficients of the terms of each equation in the system.

The Gauss-Jordan method can be used to solve a system of equations by using the following elementary row operations on an augmented matrix.

1. **Interchange two rows.**
2. **Multiply every element in a row by a nonzero real number.**
3. **Add a multiple of one row to another row.**

These operations are used to write the matrix in **reduced row echelon form**.

\[
\begin{bmatrix}
1 & 0 & | & a \\
0 & 1 & | & b
\end{bmatrix}
\]

This represents the solution, \( x = a \) and \( y = b \).

**Examples**

**Example 1**

\[
\begin{bmatrix}
1 & 2 & 5 \\
-1 & 8 & 1
\end{bmatrix}
\]

is a \( 1 \times 3 \) matrix (a row matrix).

\[
\begin{bmatrix}
4 \\
1
\end{bmatrix}
\]

is a \( 2 \times 1 \) matrix (a column matrix).

**Example 2**

The augmented matrix for

\[
\begin{align*}
x + y &= -12 \\
x - 2y &= 6
\end{align*}
\]

is

\[
\begin{bmatrix}
4 & 1 & -12 \\
1 & -2 & 6
\end{bmatrix}
\]

**Example 3**

Solve the system from Example 2 by using the Gauss-Jordan method.

1. \( R_1 \leftrightarrow R_2 \)

\[
\begin{bmatrix}
1 & -2 & | & 6 \\
4 & 1 & | & -12
\end{bmatrix}
\]

2. \(-4R_1 + R_2 \Rightarrow R_2\)

\[
\begin{bmatrix}
1 & -2 & | & 6 \\
0 & 9 & | & -36
\end{bmatrix}
\]

3. \( \frac{1}{3}R_2 \Rightarrow R_2 \)

\[
\begin{bmatrix}
1 & -2 & | & 6 \\
0 & 1 & | & -4
\end{bmatrix}
\]

4. \( 2R_2 + R_1 \Rightarrow R_1 \)

\[
\begin{bmatrix}
1 & 0 & | & -2 \\
0 & 1 & | & -4
\end{bmatrix}
\]

From the reduced row echelon form of the matrix we have \( x = -2 \) and \( y = -4 \). The solution set is \((-2, -4)\).
Chapter 3  Review Exercises

Section 3.1

1. Determine if the ordered pair is a solution to the system.

\[-5x - 7y = 4\]
\[y = \frac{1}{2}x - 1\]

a. (2, 2)  
   b. (2, -2)

For Exercises 2–4, answer true or false.

2. An inconsistent system has one solution.

3. Parallel lines form an inconsistent system.

4. Lines with different slopes intersect in one point.

For Exercises 5–8, solve the system by graphing.

5. \( f(x) = x - 1 \)
   \( g(x) = 2x - 4 \)

6. \( y = 2x + 7 \)
   \( y = -x - 5 \)

7. \( 6x + 2y = 4 \)
   \( 3x = -y + 2 \)

8. \( y = \frac{1}{2}x - 2 \)
   \( -4x + 8y = -8 \)

Section 3.2

For Exercises 9–12, solve the systems by using the substitution method.

9. \( y = \frac{3}{4}x - 4 \)
   \( -x + 2y = -6 \)

10. \( 3x = 11y - 9 \)
    \( y = \frac{3}{11}x + \frac{6}{11} \)

11. \( 4x + y = 7 \)
    \( x + \frac{1}{4}y = \frac{7}{4} \)

12. \( 6x + y = 5 \)
    \( 5x + y = 3 \)

Section 3.3

For Exercises 13–22, solve the systems by using the addition method.

13. \( \frac{2}{5}x + \frac{3}{5}y = 1 \)
    \( x - \frac{2}{3}y = \frac{1}{3} \)

14. \( 4x + 3y = 5 \)
    \( 3x - 4y = 10 \)

15. \( 3x + 4y = 2 \)
    \( 2x + 5y = -1 \)

16. \( 3x + y = 1 \)
    \( -x - \frac{1}{3}y = -\frac{1}{3} \)

17. \( 2y = 3x - 8 \)
    \( -6x = -4y + 4 \)

18. \( 3x + y = 16 \)
    \( 3(x + y) = y + 2x + 2 \)

19. \( -(y + 4x) = 2x - 9 \)
    \( -2x + 2y = -10 \)

20. \( -(4x - 3y) = 3y \)
    \( -x + 15 = y \)

21. \( -0.4x + 0.3y = 1.8 \)
    \( 0.02x - 0.01y = -0.11 \)

22. \( 0.6x - 0.2y = -1.2 \)
    \( 0.01x + 0.04y = 0.26 \)
Section 3.4

23. Melinda invested twice as much money in an account paying 5% simple interest as she did in an account paying 3.5% simple interest. If her total interest at the end of 1 yr is $303.75, find the amount she invested in the 5% account.

24. A school carnival sold tickets to ride on a Ferris wheel. The charge was $1.50 for adults and $1.00 for students. If 54 tickets were sold for a total of $70.50, how many of each type of ticket were sold?

25. How many liters of 20% saline solution must be mixed with 50% saline solution to produce 16 L of a 31.25% saline solution?

26. It takes a pilot 1 1/2 hr to travel with the wind from Jacksonville, Florida, to Myrtle Beach, South Carolina. Her return trip takes 2 hr flying against the wind. What is the speed of the wind and the speed of the plane in still air if the distance between Jacksonville and Myrtle Beach is 280 mi?

27. Two phone companies offer discount rates to students.
   Company 1: $9.95 per month, plus $0.10 per minute for long-distance calls
   Company 2: $12.95 per month, plus $0.08 per minute for long-distance calls
   a. Write a linear function describing the total cost for x min of long-distance calls from Company 1.
   b. Write a linear function describing the total cost for x min of long-distance calls from Company 2.
   c. How many minutes of long-distance calls would result in equal cost for both offers?

Section 3.5

For Exercises 29–34, graph the solution set.

29. \(2x > -y + 5\)  
30. \(2x \leq -8 - 3y\)

31. \(x > -3\)  
32. \(x \leq 2\)

33. \(x \geq \frac{1}{2}y\)  
34. \(x < \frac{2}{3}y\)
For Exercises 35–38, graph the system of inequalities.

35. \(2x - y > -2\) and \(2x - y \leq 2\)

36. \(3x + y \geq 6\) or \(3x + y < -6\)

37. \(y \geq x\) or \(y \leq -x\)

38. \(x \geq 0,\ y \geq 0,\ \text{and}\ y \leq \frac{2}{3}x + 4\)

39. Suppose a farmer has 100 acres of land on which to grow oranges and grapefruit. Furthermore, because of demand from his customers, he wants to plant at least 4 times as many acres of orange trees as grapefruit trees.

Let \(x\) represent the number of acres of orange trees.

Let \(y\) represent the number of acres of grapefruit trees.

a. Write two inequalities that express the fact that the farmer cannot use a negative number of acres to plant orange and grapefruit trees.

b. Write an inequality that expresses the fact that the total number of acres used for growing orange and grapefruit trees is at most 100.

c. Write an inequality that expresses the fact that the farmer wants to plant at least 4 times as many orange trees as grapefruit trees.

d. Sketch the inequalities in parts (a)–(c) to find the feasible region for the farmer’s distribution of orange and grapefruit trees.
Section 3.6

For Exercises 40–43, solve the systems of equations. If a system does not have a unique solution, label the system as either dependent or inconsistent.

40. \(5x + 5y + 5z = 30\)  
\(-x + y + z = 2\)  
\(10x + 6y - 2z = 4\)

41. \(5x + 3y - z = 5\)  
\(-x + 2y + z = 6\)  
\(-x - 2y - z = 8\)

42. \(x + y + z = 4\)  
\(3x + 4z = 5\)  
\(-x - 2y - 3z = -6\)  
\(2y + 3z = 2\)  
\(2x + 4y + 6z = 12\)  
\(2x - 5y = 8\)

44. The perimeter of a right triangle is 30 ft. One leg is 2 ft longer than twice the shortest leg. The hypotenuse is 2 ft less than 3 times the shortest leg. Find the lengths of the sides of this triangle.

45. Three pumps are working to drain a construction site. Working together, the pumps can drain 950 gal/hr of water. The slowest pump drains 150 gal/hr less than the fastest pump. The fastest pump drains 150 gal/hr less than the sum of the other two pumps. How many gallons can each pump drain per hour?

46. The smallest angle in a triangle measures 9° less than the middle angle. The largest angle is 26° more than 3 times the measure of the smallest angle. Find the measure of the three angles.

Section 3.7

For Exercises 47–50, determine the order of each matrix.

47. \[
\begin{bmatrix}
2 & 4 & -1 \\
5 & 0 & -3 \\
-1 & 6 & 10
\end{bmatrix}
\]

48. \[
\begin{bmatrix}
-5 & 6 \\
9 & 2 \\
0 & -3
\end{bmatrix}
\]

49. \[
\begin{bmatrix}
0 & 13 & -4 & 16
\end{bmatrix}
\]

50. \[
\begin{bmatrix}
7 \\
12 \\
-4
\end{bmatrix}
\]

For Exercises 51–52, set up the augmented matrix.

51. \(x + y = 3\)  
\(x - y = -1\)  
\(2x - y + 3z = 8\)  
\(-2x + 2y - z = -9\)

52. \(x - y + z = 4\)  
\(x - y + 3z = 8\)  
\(-2x + 2y - z = -9\)

For Exercises 53–54, write a corresponding system of equations from the augmented matrix.

53. \[
\begin{bmatrix}
1 & 0 & 0 & 9 \\
0 & 1 & -3
\end{bmatrix}
\]

54. \[
\begin{bmatrix}
1 & 0 & 0 & -5 \\
0 & 1 & 2
\end{bmatrix}
\]

55. Given the matrix \(C\)

\[
C = \begin{bmatrix}
1 & 3 & 1 \\
4 & -1 & 6
\end{bmatrix}
\]

a. What is the element in the second row and first column?

b. Write the matrix obtained by multiplying the first row by \(-4\) and adding the result to row 2.

56. Given the matrix \(D\)

\[
D = \begin{bmatrix}
1 & 2 & 0 & -3 \\
4 & -1 & 1 & 0 \\
-3 & 2 & 2 & 5
\end{bmatrix}
\]

a. Write the matrix obtained by multiplying the first row by \(-4\) and adding the result to row 2.

b. Using the matrix obtained in part (a), write the matrix obtained by multiplying the first row by 3 and adding the result to row 3.

For Exercises 57–60, solve the system by using the Gauss-Jordan method.

57. \(x + y = 3\)  
\(x - y = -1\)

58. \(4x + 3y = 6\)  
\(12x + 5y = -6\)

59. \(x - y + z = -4\)  
\(2x + y - 2z = 9\)  
\(x + 2y + z = 5\)

60. \(x - y + z = 4\)  
\(2x - y + 3z = 8\)  
\(-2x + 2y - z = -9\)
Chapter 3 Test

1. Determine if the ordered pair \((\frac{1}{2}, 2)\) is a solution to the system.
   
   \[
   \begin{align*}
   4x - 3y &= -5 \\
   12x + 2y &= 7
   \end{align*}
   \]

For Exercises 2–4, match each figure with the appropriate description, a, b, or c.

   a. The system is consistent and dependent. There are infinitely many solutions.
   b. The system is consistent and independent. There is one solution.
   c. The system is inconsistent and independent. There are no solutions.

2. [Figure]
3. [Figure]
4. [Figure]
5. Solve the system by graphing.
   
   \[
   \begin{align*}
   4x - 2y &= -4 \\
   3x + y &= 7
   \end{align*}
   \]

6. Solve the system by graphing.
   
   \[
   \begin{align*}
   f(x) &= x + 3 \\
   g(x) &= \frac{3}{2}x - 2
   \end{align*}
   \]

7. Solve the system by using the substitution method.
   
   \[
   \begin{align*}
   3x + 5y &= 13 \\
   y &= x + 9
   \end{align*}
   \]

8. Solve the system by using the addition method.
   
   \[
   \begin{align*}
   6x + 8y &= 5 \\
   3x - 2y &= 1
   \end{align*}
   \]

For Exercises 9–13, solve the system of equations.

9. \[
   \begin{align*}
   7y &= 5x - 21 \\
   9y + 2x &= -27
   \end{align*}
   \]
10. \[
   \begin{align*}
   3x - 5y &= -7 \\
   -18x + 30y &= 42
   \end{align*}
   \]
11. \[
   \begin{align*}
   \frac{1}{5}x &= \frac{1}{2}y + \frac{17}{5} \\
   \frac{1}{4}(x + 2) &= \frac{1}{6}y
   \end{align*}
   \]
12. \[
   \begin{align*}
   4x &= 5 - 2y \\
   y &= -2x + 4
   \end{align*}
   \]
13. \[
   \begin{align*}
   -0.03y + 0.06x &= 0.3 \\
   0.4x - 2 &= -0.5y
   \end{align*}
   \]
14. Graph the solution set \(2x - 5y \geq 10\).
Chapter 3  Systems of Linear Equations and Inequalities

For Exercises 15–16, graph the solution set.

15. \( x + y < 3 \) and \( 3x - 2y > -6 \)

16. \( 5x \leq 5 \) or \( x + y \leq 0 \)

17. After menopause, women are at higher risk for hip fractures as a result of low calcium. As early as their teen years, women need at least 1200 mg of calcium per day (the USDA recommended daily allowance). One 8-oz glass of skim milk contains 300 mg of calcium, and one antacid tablet (regular strength) contains 400 mg of calcium. Let \( x \) represent the number of 8-oz glasses of milk that a woman drinks per day. Let \( y \) represent the number of antacid tablets (regular strength) that a woman takes per day.

a. Write two inequalities that express the fact that the number of glasses of milk and the number of antacid tablets taken each day cannot be negative.

b. Write a linear inequality in terms of \( x \) and \( y \) for which the daily calcium intake is at least 1200 mg.

c. Graph the inequalities.

For Exercises 18–19, solve the system.

18. \( 2x + 2y + 4z = -6 \)
   \( 3x + y + 2z = 29 \)
   \( x - y - z = 44 \)

19. \( 2(x + z) = 6 + x - 3y \)
   \( 2x = 11 + y - z \)
   \( x + 2(y + z) = 8 \)

20. How many liters of a 20% acid solution should be mixed with a 60% acid solution to produce 200 L of a 44% acid solution?

21. Two angles are complementary. Two times the measure of one angle is less than the measure of the other. Find the measure of each angle.

22. Working together, Joanne, Kent, and Geoff can process 504 orders per day for their business. Kent can process 20 more orders per day than Joanne can process. Geoff can process 10 fewer orders per day than Kent and Joanne combined. Find the number of orders that each person can process per day.

23. Write an example of a 3 \( \times \) 2 matrix.

24. Given the matrix \( A \)

\[
A = \begin{bmatrix}
1 & 2 & 1 \\
4 & 0 & 1 \\
-5 & -6 & 3
\end{bmatrix}
\]

a. Write the matrix obtained by multiplying the first row by \ (-4 \) and adding the result to row 2.

b. Using the matrix obtained in part (a), write the matrix obtained by multiplying the first row by \ (5 \) and adding the result to row 3.

For Exercises 25–26, solve by using the Gauss-Jordan method.

25. \( 5x - 4y = 34 \)
   \( x - 2y = 8 \)

26. \( x + y + z = 1 \)
   \( 2x + y = 0 \)
   \( -2y - z = 5 \)
Cumulative Review Exercises

1. Simplify. $2[3^2 - 8(7 - 5)]$

2. Simplify. $7[4 - 2(w - 5) - 3(2w + 1)] + 20$

For Exercises 3–5, solve the equation.

3. $-5(2x - 1) - 2(3x + 1) = 7 - 2(8x + 1)$

4. $\frac{1}{2}(a - 2) - \frac{3}{4}(2a + 1) = -\frac{1}{6}$

5. $-4 = |2x - 3| - 9$

For Exercises 6–11, solve the inequality. Write the answer in interval notation if possible.

6. $-3y - 2(y + 1) < 8$

7. $4x > 16$ or $-6x - 3 \geq 9$

8. $4x > 16$ and $-6x - 3 \geq 9$

9. $0 \leq \frac{3x - 9}{6} \leq 5$

10. $|x - 4| + 1 < 11$

11. $4 < |2x + 4|$

12. Graph the solution set. $x - 5y \leq 5$

13. Identify the slope and the $x$- and $y$-intercepts of the line $5x - 2y = 15$.

14. $y = -\frac{1}{3}x - 4$

15. $x = -2$

16. Find the slope of the line passing through the points $(4, -10)$ and $(6, -10)$.

17. Find an equation for the line that passes through the points $(3, -8)$ and $(2, -4)$. Write the answer in slope-intercept form.

18. Solve the system by using the addition method.

\[
\begin{align*}
2x - 3y &= 6 \\
\frac{1}{2}x - \frac{3}{4}y &= 1
\end{align*}
\]

19. Solve the system by using the substitution method.

\[
\begin{align*}
2x + y &= 4 \\
y &= 3x - 1
\end{align*}
\]

20. A child’s piggy bank contains 19 coins consisting of nickels, dimes, and quarters. The total amount of money in the bank is $3.05. If the number of quarters is 1 more than twice the number of nickels, find the number of each type of coin in the bank.

21. Two video clubs rent DVDs according to the following fee schedules:

Club 1: $25 initiation fee plus $2.50 per DVD

Club 2: $10 initiation fee plus $3.00 per DVD

a. Write a linear function describing the total cost of renting $x$ DVDs from club 1.

b. Write a linear function describing the total cost of renting $x$ DVDs from club 2.

c. How many DVDs would have to be rented to make the cost for club 1 the same as the cost for club 2?
22. Solve the system.
\[
\begin{align*}
3x & = 3 - 2y - 3z \\
4x - 5y + 7z & = 1 \\
2x + 3y - 2z & = 6
\end{align*}
\]

23. Determine the order of the matrix.
\[
\begin{bmatrix}
4 & 5 & 1 \\
-2 & 6 & 0
\end{bmatrix}
\]

24. Write an example of a $2 \times 4$ matrix.

25. Solve the system by using the Gauss-Jordan method.
\[
\begin{align*}
2x - 4y & = -2 \\
4x + y & = 5
\end{align*}
\]