Linear Equations and Inequalities in One Variable

CHAPTER OUTLINE

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Chapter 1

In this chapter, we study linear equations and inequalities and their applications.

Are You Prepared?

To prepare yourself, try the crossword puzzle. The clues in the puzzle review formulas from geometry and other important mathematical facts that you will encounter in this chapter as you work through the application problems.

Across
2. What is the next consecutive integer after 1306?
4. Given that distance = rate \times time, what is the distance between Atlanta and Los Angeles if it takes 33 hr, traveling 60 mph?
6. What is the next consecutive odd integer after 7803?
7. What is 10% of 64,780?
9. If an angle measures 41°, what is the complement?
10. If an angle measures 70°, what is the supplement?

Down
1. What is 40% of 32,640?
2. What is the sum of the measures of the angles in a triangle?
3. Evaluate \left| -7729 + 262 \right|.
4. What number is 50 more than twice 8707?
5. If the area of a rectangle is 6370 ft², and the width is 65 ft, what is the length?
8. What is the amount of simple interest earned on $4000 at 5% interest for 4 yr?
## Linear Equations in One Variable

### 1. Definition of a Linear Equation in One Variable

An **equation** is a statement that indicates that two quantities are equal. The following are examples of equations:

\[
\begin{align*}
x & = -4 \\
p + 3 & = 11 \\
-2z & = -20
\end{align*}
\]

All equations have an equal sign. Furthermore, notice that the equal sign separates the equation into two parts, the left-hand side and the right-hand side. A **solution to an equation** is a value of the variable that makes the equation a true statement. Substituting a solution to an equation for the variable makes the right-hand side equal to the left-hand side.

**Equation** | **Solution** | **Check**
--- | --- | ---
\(p + 3 = 11\) | 8 | Substitute 8 for \(p\).
\(8 + 3 = 11 \checkmark\) Right-hand side equals left-hand side.
\(-2z = -20\) | 10 | Substitute 10 for \(z\).
\(-2(10) = -20 \checkmark\) Right-hand side equals left-hand side.

The **solution set** to an equation is the set of all solutions to an equation. We write the solution set using set brackets. For example:

**Equation** | **Solution set**
--- | ---
\(p + 3 = 11\) | \(\{8\}\) This equation has one solution.
\(w^2 = 16\) | \(\{4, -4\}\) This equation has two solutions.

Throughout this text we will learn to recognize and solve several different types of equations, but in this chapter, we will focus on the specific type of equation called a linear equation in one variable.

**Definition**  
A **linear equation in one variable** is an equation that can be written in the form

\[ax + b = 0\]

Notice that a linear equation in one variable will contain only one variable. Furthermore, because the variable has an implied exponent of 1, a linear equation is sometimes called a first-degree equation.

**Linear equation in one variable** | **Not a linear equation in one variable**
--- | ---
\(4x - 3 = 0\) | \(4x^2 + 8 = 0\) (exponent for \(x\) is not 1)
\(\frac{1}{2}p + \frac{3}{10} = 0\) | \(\frac{1}{2}p + \frac{3}{10}q = 0\) (more than one variable)
\(5x + 2 = 0\) | \(5\sqrt{x} + 2 = 0\) (Equation is not in the form \(ax + b = 0\). The variable is in the denominator.)
2. Solving Linear Equations

To solve a linear equation, the goal is to simplify the equation to isolate the variable. Each step used in simplifying an equation results in an equivalent equation. Equivalent equations have the same solution set. For example, the equations $2x + 3 = 7$ and $2x = 4$ are equivalent because $\{2\}$ is the solution set for both equations.

To solve an equation, we may use the addition, subtraction, multiplication, and division properties of equality. These properties state that adding, subtracting, multiplying, or dividing the same quantity on each side of an equation results in an equivalent equation.

**PROPERTY Addition and Subtraction Properties of Equality**

Let $a$, $b$, and $c$ represent real numbers.

Addition property of equality: If $a = b$, then $a + c = b + c$.

*Subtraction property of equality: If $a = b$, then $a - c = b - c$.

*The subtraction property of equality follows directly from the addition property, because subtraction is defined in terms of addition.

\[
\text{If } a + (-c) = b + (-c) \\
\text{then, } a - c = b - c
\]

**PROPERTY Multiplication and Division Properties of Equality**

Let $a$, $b$, and $c$ represent real numbers.

Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$.

*Division property of equality: If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ (provided $c \neq 0$).

*The division property of equality follows directly from the multiplication property, because division is defined as multiplication by the reciprocal.

\[
\text{If } a \cdot \frac{1}{c} = b \cdot \frac{1}{c} (c \neq 0) \\
\text{then, } \frac{a}{c} = \frac{b}{c}
\]

**Example 1** Solving a Linear Equation

Solve the equation. \[12 + x = 40\]

**Solution:**

\[
12 + x = 40 \\
12 - 12 + x = 40 - 12 \\
x = 28
\]

To isolate $x$, subtract 12 from both sides. Simplify.
Chapter 1  Linear Equations and Inequalities in One Variable

Check:  
\[ 12 + x = 40 \]
\[ 12 + (28) \neq 40 \]
\[ 40 \neq 40 \checkmark \text{ True.} \]

The solution set is \( \{28\} \).

Skill Practice  Solve the equation.

1. \( x - 5 = -11 \)

Example 2  Solving Linear Equations

Solve each equation.

a. \( \frac{3}{5}p = \frac{4}{15} \)

Solution:

a.  
\[ \frac{3}{5}p = \frac{4}{15} \]
\[ \left( \frac{5}{3} \right) \left( \frac{3}{5}p \right) = \left( \frac{5}{3} \right) \left( \frac{4}{15} \right) \]  
To isolate \( p \), multiply both sides by the reciprocal of \( \frac{3}{5} \).
\[ p = \left( \frac{5}{3} \right) \left( \frac{4}{15} \right) \]
\[ p = \frac{4}{9} \]

The value \( \frac{4}{9} \) checks in the original equation.

The solution set is \( \left\{ -\frac{4}{9} \right\} \).

b.  
\[ 4 = \frac{w}{2.2} \]

b.  
\[ 2.2(4) = \left( \frac{w}{2.2} \right) \cdot 2.2 \]
\[ 8.8 = w \]

The value 8.8 checks in the original equation.

The solution set is \( \{8.8\} \).

c.  
\[ -x = 6 \]
\[ -1(-x) = -1(6) \]
\[ x = -6 \]

The value \( -6 \) checks in the original equation.

The solution set is \( \{-6\} \).

Skill Practice  Solve the equations.

2. \( \frac{6}{5}y = -\frac{3}{5} \)

3. \( 5 = \frac{t}{16} \)

4. \( -a = -2 \)
Section 1.1 Linear Equations in One Variable

For more complicated linear equations, several steps are required to isolate the variable. These steps are listed below.

**PROCEDURE Solving a Linear Equation in One Variable**

**Step 1** Simplify both sides of the equation.
- Clear parentheses.
- Consider clearing fractions or decimals (if any are present) by multiplying both sides of the equation by a common denominator of all terms.
- Combine like terms.

**Step 2** Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.

**Step 3** Use the addition or subtraction property of equality to collect the constant terms on the other side of the equation.

**Step 4** Use the multiplication or division property of equality to make the coefficient of the variable term equal to 1.

**Step 5** Check your answer and write the solution set.

---

**Example 3 Solving a Linear Equation**

Solve the linear equation and check the answer.

\[ 3x + 1 = -7 \]

**Solution:**

\[
\begin{align*}
3x + 1 &= -7 \\
3x + 1 - 1 &= -7 - 1 \\
3x &= -8 \\
\frac{3x}{3} &= \frac{-8}{3} \\
x &= -\frac{8}{3}
\end{align*}
\]

To isolate \( x \), divide both sides of the equation by 3.

**Check:**

\[
3 \left( -\frac{8}{3} \right) + 1 \overset{?}{=} -7 \\
-8 + 1 \overset{?}{=} -7 \\
-7 \overset{\checkmark}{=} -7 \text{ True.}
\]

The solution set is \( \left\{ -\frac{8}{3} \right\} \).

**Skill Practice** Solve the linear equation and check the answer.

5. \( 5x - 19 = -23 \)

**Answer**

5. \( \left\{ -\frac{4}{5} \right\} \)
Chapter 1  Linear Equations and Inequalities in One Variable

Example 4  Solving a Linear Equation

Solve the linear equation and check the answer.

$$11z + 2 = 5(z - 2)$$

Solution:

$$11z + 2 = 5(z - 2)$$

Apply the distributive property to clear parentheses.

$$11z - 5z + 2 = 5z - 5z - 10$$

Subtract 5z from both sides.

$$6z + 2 = -10$$

Combine like terms.

$$6z + 2 - 2 = -10 - 2$$

Subtract 2 from both sides.

$$6z = -12$$

Combine like terms.

$$\frac{6z}{6} = \frac{-12}{6}$$

To isolate z, divide both sides of the equation by 6.

$$z = -2$$

Simplify.

Check:

$$11z + 2 = 5(z - 2)$$

$$11(-2) + 2 \neq 5(-2 - 2)$$

$$-22 + 2 \neq 5(-4)$$

$$-20 \neq -20 \checkmark$$  True.

The solution set is \(-2\).

Skill Practice  Solve the equations.

6.  \(7 + 2(y - 3) = 6y + 3\)

Example 5  Solving a Linear Equation

Solve the equation.  

$$-3(x - 4) + 2 = 7 - (x + 1)$$

Solution:

$$-3(x - 4) + 2 = 7 - (x + 1)$$

Clear parentheses.

$$-3x + 12 + 2 = 7 - x - 1$$

Combine like terms.

$$-3x + 14 = -x + 6$$

Add x to both sides of the equation.

$$-3x + x + 14 = -x + x + 6$$

Combine like terms.

$$-2x + 14 = 6$$

Subtract 14 from both sides.

Answer

6.  \(\left\{\frac{-1}{2}\right\}\)
Section 1.1  Linear Equations in One Variable

-2x = -8  
\[ \frac{-2x}{-2} = \frac{-8}{-2} \]
\[ x = 4 \]

Combine like terms.
To isolate \( x \), divide both sides by -2.
Simplify. The solution checks in the original equation.

The solution set is \( \{4\} \).

**Skill Practice** Solve the equation.
7. \( 4(2t + 2) - 6(t - 1) = 6 - t \)

---

**Example 6** Solving a Linear Equation

Solve the equation. \(-4[y - 3(y - 5)] = 2(6 - 5y)\)

**Solution:**
\[-4[y - 3(y - 5)] = 2(6 - 5y)\]
\[-4[y - 3y + 15] = 12 - 10y\]
\[-4[-2y + 15] = 12 - 10y\]
\[8y - 60 = 12 - 10y\]
\[8y + 10y - 60 = 12 - 10y + 10y\]
\[18y - 60 = 12\]
\[18y - 60 + 60 = 12 + 60\]
\[18y = 72\]
\[\frac{18y}{18} = \frac{72}{18}\]
\[y = 4\]

To isolate \( y \), divide both sides by 18.
The solution checks.

The solution set is \( \{4\} \).

**Skill Practice** Solve the equation.
8. \( 3[p + 2(p - 2)] = 4(p - 3) \)

---

3. Clearing Fractions and Decimals

When an equation contains fractions or decimals, it is sometimes helpful to clear the fractions and decimals. This is accomplished by multiplying both sides of the equation by the least common denominator (LCD) of all terms within the equation. This is demonstrated in Example 7.

**Answers**
7. \( \{ \frac{8}{3} \} \)  
8. \( \{0\} \)
Example 7  Solving a Linear Equation by Clearing Fractions

Solve the equation.
\[ \frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}(w - 4) \]

Solution:
\[ \frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}(w - 4) \]
Clear parentheses.
\[ 12 \cdot \left( \frac{1}{4}w + \frac{1}{3}w - 1 \right) = 12 \cdot \left( \frac{1}{2}w - 2 \right) \]
Multiply both sides of the equation by the LCD of all terms. In this case, the LCD is 12.
\[ 12 \cdot \frac{1}{4}w + 12 \cdot \frac{1}{3}w + 12 \cdot (-1) = 12 \cdot \frac{1}{2}w + 12 \cdot (-2) \]
Apply the distributive property.
\[ 3w + 4w - 12 = 6w - 24 \]
\[ 7w - 12 = 6w - 24 \]
\[ w - 12 = -24 \]
\[ w = -12 \]
The solution checks.
The solution set is \{-12\}.

Skill Practice  Solve the equation by first clearing the fractions.
9. \( \frac{3}{4}a + \frac{1}{2} = \frac{2}{3}a + \frac{1}{3} \)

Example 8  Solving a Linear Equation by Clearing Fractions

Solve.
\[ \frac{x - 2}{5} - \frac{x - 4}{2} = 2 + \frac{x + 4}{10} \]

Solution:
\[ \frac{x - 2}{5} - \frac{x - 4}{2} = 2 + \frac{x + 4}{10} \]
The LCD of all terms in the equation is 10.
\[ 10 \left( \frac{x - 2}{5} - \frac{x - 4}{2} \right) = 10 \left( \frac{2}{1} + \frac{x + 4}{10} \right) \]
Multiply both sides by 10.
\[ \frac{2}{1} \cdot \left( \frac{x - 2}{5} \right) - \frac{5}{1} \cdot \left( \frac{x - 4}{2} \right) = \frac{10}{1} \cdot \left( \frac{2}{1} \right) + \frac{10}{1} \cdot \left( \frac{x + 4}{10} \right) \]
Apply the distributive property.

Answer
9. \{-2\}
Section 1.1 Linear Equations in One Variable

Clear fractions.
Apply the distributive property.
Simplify both sides of the equation.
Subtract $x$ from both sides.
Subtract 16 from both sides.
The value $-2$ checks in the original equation.

The solution set is \{-2\}.

Skill Practice Solve.
10. \( \frac{1}{8} - \frac{x + 3}{4} = \frac{3x - 2}{2} \)

The same procedure used to clear fractions in an equation can be used to clear decimals.

Example 9 Solving a Linear Equation by Clearing Decimals

Solve the equation. \( 0.55x - 0.6 = 2.05x \)

Solution:
Recall that any terminating decimal can be written as a fraction. Therefore, the equation \( 0.55x - 0.6 = 2.05x \) is equivalent to

\[
\frac{55}{100}x - \frac{6}{10} = \frac{205}{100}x
\]

A convenient common denominator for all terms in this equation is 100. Multiplying both sides of the equation by 100 will have the effect of “moving” the decimal point 2 places to the right.

\[
100(0.55x - 0.6) = 100(2.05x) \quad \text{Multiply both sides by 100 to clear decimals.}
\]

\[
55x - 60 = 205x
\]

\[
-60 = 150x \quad \text{Subtract 55x from both sides.}
\]

\[
\frac{-60}{150} = x \quad \text{To isolate } x, \text{ divide both sides by 150.}
\]

\[
x = \frac{-2}{5} = -0.4 \quad \text{The solution checks.}
\]

The solution set is \{-0.4\}.

Skill Practice Solve the equation by first clearing the decimals.
11. \( 2.2x + 0.5 = 1.6x + 0.2 \)

Answers
10. \( \frac{3}{14} \) 11. \{-0.5\}
4. Conditional Equations, Contradictions, and Identities

The solution to a linear equation is the value of \( x \) that makes the equation a true statement. While linear equations have one unique solution, some equations have no solution, and others have infinitely many solutions.

I. Conditional Equations

An equation that is true for some values of the variable but false for other values is called a conditional equation. The equation \( x + 4 = 6 \) is a conditional equation because it is true on the condition that \( x = 2 \). For other values of \( x \), the statement \( x + 4 = 6 \) is false.

II. Contradictions

Some equations have no solution, such as \( x + 1 = x + 2 \). There is no value of \( x \) that when increased by 1 will equal the same value increased by 2. If we tried to solve the equation by subtracting \( x \) from both sides, we get the contradiction \( 1 = 2 \).

\[
\begin{align*}
x + 1 &= x + 2 \\
x - x + 1 &= x - x + 2 \\
1 &= 2 \quad \text{(contradiction)}
\end{align*}
\]

This indicates that the equation has no solution. An equation that has no solution is called a contradiction. The solution set for a contradiction is the empty set and is denoted by the symbol \( \emptyset \) or \( \varnothing \).

III. Identities

An equation that is true for all real numbers is called an identity. For example, consider the equation \( x + 4 = x + 4 \). Because the left- and right-hand sides are identical, any real number substituted for \( x \) will result in equal quantities on both sides. If we solve the equation, we get the identity \( 4 = 4 \). In such a case, the solution set is the set of real numbers.

\[
\begin{align*}
x + 4 &= x + 4 \\
x - x + 4 &= x - x + 4 \\
4 &= 4 \quad \text{(identity)}
\end{align*}
\]

The solution set is the set of real numbers. In set-builder notation, we have, \( \{ x \mid x \text{ is a real number} \} \).

Example 10  Identifying Conditional Equations, Contradictions, and Identities

Identify each equation as a conditional equation, a contradiction, or an identity. Then give the solution set.

a. \( 3[x - (x + 1)] = -2 \)

b. \( 5(3 + c) + 2 = 2c + 3c + 17 \)

c. \( 4x - 3 = 17 \)
Section 1.1 Linear Equations in One Variable

Solution:

a. \(3[x - (x + 1)] = -2\)
\[3[x - x - 1] = -2\] Clear parentheses.
\[3[-1] = -2\] Combine like terms.
\[-3 = -2\] Contradiction

This equation is a contradiction. The solution set is \{ \}.

b. \(5(3 + c) + 2 = 2c + 3c + 17\)
\[15 + 5c + 2 = 5c + 17\] Clear parentheses and combine like terms.
\[5c + 17 = 5c + 17\] Identity
\[0 = 0\]

This equation is an identity. The solution set is \( \{x \mid x \text{ is a real number}\} \).

c. \(4x - 3 = 17\)
\[4x - 3 + 3 = 17 + 3\] Add 3 to both sides.
\[4x = 20\]
\[\frac{4x}{4} = \frac{20}{4}\] To isolate \(x\), divide both sides by 4.
\[x = 5\]

This equation is a conditional equation. The solution set is \{5\}.

Skill Practice Identify each equation as a conditional equation, an identity, or a contradiction. Then give the solution set.

12. \(2(-5x - 1) = 2x - 12x + 6\)
13. \(2(3x - 1) = 6(x + 1) - 8\)
14. \(4x + 1 - x = 6x - 2\)

Answers
12. Contradiction; \{ \}
13. Identity; \( \{x \mid x \text{ is a real number}\} \)
14. Conditional equation; \{1\}

Study Skills Exercises

1. Some instructors allow the use of calculators. Does your instructor allow the use of a calculator? If so, what kind?
Will you be allowed to use a calculator on tests or just for occasional calculator problems in the text?
Helpful Hint: If you are not permitted to use a calculator on tests, you should do your homework in the same way, without the calculator.

2. Define the key terms.
   a. Equation
   b. Solution to an equation
   c. Linear equation in one variable
   d. Solution set
   e. Conditional equation
   f. Contradiction
   g. Empty set
   h. Identity
Review Exercises

For Exercises 3–6, clear parentheses and combine like terms.

3. \(8x - 3y + 2xy - 5x + 12xy\)  
4. \(5ab + 5a - 13 - 2a + 17\)  
5. \(2(3z - 4) - (z + 12)\)  
6. \(- (6w - 5) + 3(4w - 5)\)

Concept 1: Definition of a Linear Equation in One Variable

For Exercises 7–12, label the equation as linear or nonlinear.

7. \(2x + 1 = 5\)  
8. \(10 = x + 6\)  
9. \(x^2 + 7 = 9\)  
10. \(3 + x^3 - x = 4\)  
11. \(-3 = x\)  
12. \(5.2 - 7x = 0\)

13. Use substitution to determine which value is the solution to \(2x - 1 = 5\).
   a. 2  
   b. 3  
   c. 0  
   d. -1

14. Use substitution to determine which value is the solution to \(2y - 3 = -2\).
   a. 1  
   b. \(\frac{1}{2}\)  
   c. 0  
   d. \(-\frac{1}{2}\)

Concept 2: Solving Linear Equations

For Exercises 15–44, solve the equation and check the solution. (See Examples 1–6.)

15. \(x + 7 = 19\)  
16. \(-3 + y = -28\)  
17. \(-x = 2\)  
18. \(-t = \frac{3}{4}\)

19. \(\frac{7}{8} = \frac{5}{6}x\)  
20. \(\frac{-12}{13} = \frac{4}{3}b\)  
21. \(\frac{a}{5} = -8\)  
22. \(\frac{x}{8} = \frac{1}{2}\)

23. \(2.53 = -2.3t\)  
24. \(-4.8 = 6.1 + y\)  
25. \(p - 2.9 = 3.8\)  
26. \(-4.2a = 4.944\)

27. \(6q - 4 = 62\)  
28. \(2w - 15 = 15\)  
29. \(4y - 17 = 35\)  
30. \(6z - 25 = 83\)

31. \(-b - 5 = 2\)  
32. \(6 = -y + 1\)  
33. \(3(x - 6) = 2x - 5\)  
34. \(13y + 4 = 5(y - 4)\)

35. \(6 - (t + 2) = 5(3t - 4)\)  
36. \(1 - 5(p + 2) = 2(p + 13)\)

37. \(6(a + 3) - 10 = -2(a - 4)\)  
38. \(8(b - 2) + 3b = -9(b - 1)\)

39. \(-2[5 - (2z + 1)] - 4 = 2(3 - z)\)  
40. \(3[w - (10 - w)] = 7(w + 1)\)

41. \(6(-y + 4) - 3(2y - 3) = -y + 5 + 5y\)  
42. \(13 + 4w = -5(-w - 6) + 2(w + 1)\)

43. \(14 - 2x + 5x = -4(-2x - 5) - 6\)  
44. \(8 - (p + 2) + 6p + 7 = p + 13\)
Section 1.1 Linear Equations in One Variable

**Concept 3: Clearing Fractions and Decimals**
For Exercises 45–56, solve the equations. (See Examples 7–9.)

45. \(\frac{2}{3}x - \frac{1}{6} = \frac{5}{12}x + \frac{3}{2} - \frac{1}{6}\)
46. \(-\frac{2}{5} + 4 = \frac{9}{10}x + \frac{2}{5}\)
47. \(\frac{1}{5}(p - 5) = \frac{3}{5}p + \frac{1}{10}p + 1\)
48. \(\frac{5}{6}(q + 2) = \frac{7}{9}q - \frac{1}{3} + 2\)
49. \(\frac{3x - 7}{2} + \frac{3 - 5x}{3} = \frac{3 - 6x}{5}\)
50. \(\frac{2y - 4}{5} = \frac{5y + 13}{4} + \frac{y}{2}\)
51. \(\frac{4}{3}(2q + 6) - \frac{5q - 6}{6} - \frac{q}{3} = 0\)
52. \(-\frac{3a + 9}{15} - \frac{2a - 5}{5} - \frac{a + 2}{10} = 0\)
53. \(6.3w - 1.5 = 4.8\)
54. \(0.2x + 53.6 = x\)
55. \(0.75(m - 2) + 0.25m = 0.5\)
56. \(0.4(n + 10) + 0.6n = 2\)

**Concept 4: Conditional Equations, Contradictions, and Identities**
57. What is a conditional equation?
58. Explain the difference between a contradiction and an identity.

For Exercises 59–64, identify the equation as a conditional equation, a contradiction, or an identity. Then give the solution set. (See Example 10.)

59. \(4x + 1 = 2(2x + 1) - 1\)
60. \(3x + 6 = 3x\)
61. \(-11x + 4(x - 3) = -2x - 12\)
62. \(5(x + 2) - 7 = 3\)
63. \(2x - 4 + 8x = 7x - 8 + 3x\)
64. \(-7x + 8 + 4x = -3(x - 3) - 1\)

**Mixed Exercises**
For Exercises 65–96, solve the equations.

65. \(-5b + 9 = -71\)
66. \(-3x + 18 = -66\)
67. \(16 = -10 + 13x\)
68. \(15 = -12 + 9x\)
69. \(10c + 3 = -3 + 12c\)
70. \(2w + 21 = 6w - 7\)
71. \(12b - 15b - 8 + 6 = 4b + 6 - 1\)
72. \(4x + 2 - 3z + 5 = 3 + z + 4\)
73. \(5(x - 2) - 2x = 3x + 7\)
74. \(2x + 3(x - 5) = 15\)
75. \(\frac{c}{2} - \frac{c}{4} + \frac{3c}{8} = 1\)
76. \(\frac{d}{5} - \frac{d}{10} + \frac{5d}{20} = \frac{7}{10}\)
77. \(0.75(8x - 4) = \frac{2}{3}(6x - 9)\)
78. \(-\frac{1}{2}(4z - 3) = -z\)
79. \(7(p + 2) - 4p = 3p + 14\)
80. \(6(z - 2) = 3z - 8 + 3z\)
81. \(4[3 + 5(3 - b) + 2b] = 6 - 2b\)
82. \(\frac{1}{3}(x + 3) - \frac{1}{6} = \frac{1}{2}(2x + 5)\)
83. \(3 - \frac{3}{4}x = 9\)
84. \(\frac{9}{10} - 4w = \frac{5}{2}\)
85. \(\frac{5}{4} + \frac{y - 3}{8} = \frac{2y + 1}{2}\)
86. \(\frac{2}{3} - \frac{x + 2}{6} = \frac{5x - 2}{2}\)
87. \(\frac{2y - 9}{10} + \frac{3}{2} = y\)
88. \(\frac{2}{3}x - \frac{5}{6}x - 3 = \frac{1}{2}x - 5\)
89. \(0.48x - 0.08x = 0.12(260 - x)\)
90. \(0.07w + 0.06(140 - w) = 90\)
91. \(0.5x + 0.25 = \frac{1}{3}x + \frac{5}{4}\)
Chapter 1  Linear Equations and Inequalities in One Variable

92. \( \frac{0.2b + \frac{1}{3}}{15} \)  
93. \( 0.3b - 1.5 = 0.25(b + 2) \)  
94. \( 0.7(a - 1) = 0.25 + 0.7a \)

95. \( \frac{-7y + \frac{1}{4}}{2(5 - \frac{3}{4}y)} \)  
96. \( 5x - (8 - x) = 2[-4 - (3 + 5x) - 13] \)

Expanding Your Skills

97. a. Simplify the expression. \(-2(y - 1) + 3(y + 2)\)  
   b. Solve the equation. \(-2(y - 1) + 3(y + 2) = 0\)  
   c. Explain the difference between simplifying an expression and solving an equation.

98. a. Simplify the expression. \(4w - 8(2 + w)\)  
   b. Solve the equation. \(4w - 8(2 + w) = 0\)  
   c. Explain the difference between simplifying an expression and solving an equation.

Problem Recognition Exercises

Equations Versus Expressions

For Exercises 1–20, identify each exercise as an expression or an equation. Then simplify the expressions and solve the equations.

1. \( 4x - 2 + 6 - 8x \)  
2. \( -3y - 3 - 4y + 8 \)  
3. \( 7b - 1 = 2b + 4 \)

4. \( 10t + 2 = 2 - 7t \)  
5. \( 4(b - 8) - 7(2b + 1) \)  
6. \( 10(2x + 3) - 8(5 - x) \)

7. \( 7(2 - x) = 5s + 8 \)  
8. \( 15(3 - 2y) = 21 + 2y \)  
9. \( 2(3x - 4) - 4(5x + 1) = -8x + 7 \)

10. \( 6(2 - 3a) - 2(8a + 3) = -12a - 19 \)  
11. \( \frac{1}{2}y + \frac{3}{5} - \frac{2}{3}y - \frac{7}{10} \)

12. \( -\frac{7}{8} - \frac{4}{3}t - \frac{5}{4} + \frac{11}{6}u \)  
13. \( 20x - 8 + 7x + 28 = 27x - 9 \)  
14. \( 7 + 8b - 12 = 3b - 8 + 5b \)

15. \( \frac{5}{6}y - \frac{7}{8} = \frac{1}{2}y + \frac{3}{4} \)  
16. \( \frac{4}{5} + 3z = \frac{1}{2}z + 1 \)  
17. \( 0.29c + 4.495 - 0.12c \)

18. \( 0.45k - 1.67 + 0.89 - 1.456k \)  
19. \( 0.125(2p - 8) = 0.25(p - 4) \)

20. \( 0.5u + 1.2 - 0.74u = 0.8 - 0.24u + 0.4 \)
Applications of Linear Equations in One Variable

1. Introduction to Problem Solving

One of the important uses of algebra is to develop mathematical models for understanding real-world phenomena. To solve an application problem, relevant information must be extracted from the wording of a problem and then translated into mathematical symbols. This is a skill that requires practice. The key is to stick with it and not to get discouraged.

Problem-Solving Flowchart for Word Problems

Step 1
- Read the problem carefully.
  • Familiarize yourself with the problem. Estimate the answer, if possible.

Step 2
- Assign labels to unknown quantities.
  • Identify the unknown quantity or quantities. Let \( x \) represent one of the unknowns. Draw a picture and write down relevant formulas.

Step 3
- Develop a verbal model.
  • Write an equation in words.

Step 4
- Write a mathematical equation.
  • Replace the verbal model with a mathematical equation using \( x \) or another variable.

Step 5
- Solve the equation.
  • Solve for the variable, using the steps for solving linear equations.

Step 6
- Interpret the results and write the final answer in words.
  • Once you’ve obtained a numerical value for the variable, recall what it represents in the context of the problem. Can this value be used to determine other unknowns in the problem? Write an answer to the word problem in words.

To write an English statement as an algebraic expression, review the list of key terms given in Table 1-1.

<table>
<thead>
<tr>
<th><strong>Addition:</strong> ( a + b )</th>
<th><strong>Subtraction:</strong> ( a - b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>the sum of ( a ) and ( b )</td>
<td>the difference of ( a ) and ( b )</td>
</tr>
<tr>
<td>( a ) plus ( b )</td>
<td>( a ) minus ( b )</td>
</tr>
<tr>
<td>( b ) added to ( a )</td>
<td>( b ) subtracted from ( a )</td>
</tr>
<tr>
<td>( b ) more than ( a )</td>
<td>( a ) decreased by ( b )</td>
</tr>
<tr>
<td>( a ) increased by ( b )</td>
<td>( b ) less than ( a )</td>
</tr>
<tr>
<td>the total of ( a ) and ( b )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Multiplication:</strong> ( a \times b )</th>
<th><strong>Division:</strong> ( a \div b, \frac{a}{b} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>the product of ( a ) and ( b )</td>
<td>the quotient of ( a ) and ( b )</td>
</tr>
<tr>
<td>( a ) times ( b )</td>
<td>( a ) divided by ( b )</td>
</tr>
<tr>
<td>( a ) multiplied by ( b )</td>
<td>( b ) divided into ( a )</td>
</tr>
<tr>
<td>the ratio of ( a ) and ( b )</td>
<td>( a ) over ( b )</td>
</tr>
<tr>
<td>( a ) per ( b )</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 1 Linear Equations and Inequalities in One Variable

Example 1 Translating and Solving a Linear Equation

The sum of two numbers is 39. One number is 3 less than twice the other. What are the numbers?

Solution:

Step 1: Read the problem carefully.

Step 2: Let \( x \) represent one number.
Let \( 2x - 3 \) represent the other number.

Step 3: (One number) + (other number) = 39

Step 4: Replace the verbal model with a mathematical equation.

\[
\begin{align*}
(x) + (2x - 3) &= 39 \\
3x - 3 &= 39 \\
3x &= 42 \\
x &= \frac{42}{3} \\
x &= 14 \\
\end{align*}
\]

Step 5: Solve for \( x \).

Step 6: Interpret your results. Refer back to step 2.

One number is \( x \): \( 14 \)
The other number is \( 2x - 3 \): \( 2(14) - 3 \)

Answer: The numbers are 14 and 25.

Skill Practice

1. One number is 5 more than 3 times another number. The sum of the numbers is 45. Find the numbers.

2. Applications Involving Consecutive Integers

The word consecutive means “following one after the other in order.”

- The numbers \(-2, -1, 0, 1, 2, \) and so on are examples of consecutive integers. Notice that two consecutive integers differ by 1. Therefore, if \( x \) represents an integer, then \( x + 1 \) represents the next consecutive integer.

- The numbers \( 2, 4, 6, 8, \) and so on are consecutive even integers. Consecutive even integers differ by 2. Therefore, if \( x \) represents an even integer, then \( x + 2 \) represents the next consecutive even integer.

- The numbers \( 15, 17, 19, \) and so on are consecutive odd integers. Consecutive odd integers also differ by 2. Therefore, if \( x \) represents an odd integer, then \( x + 2 \) represents the next consecutive odd integer.

Answer

1. The numbers are 10 and 35.
Example 2 Solving a Linear Equation Involving Consecutive Integers

Three times the sum of two consecutive odd integers is 516. Find the integers.

Solution:

Step 1: Read the problem carefully.

Step 2: Label the unknown:
Let \( x \) represent the first odd integer.
Then \( x + 2 \) represents the next odd integer.

Step 3: Write an equation in words.

\[ 3[(\text{first odd integer}) + (\text{second odd integer})] = 516 \]
\[ 3[x + (x + 2)] = 516 \]
\[ 3(2x + 2) = 516 \]
\[ 6x + 6 = 516 \]
\[ 6x = 510 \]
\[ x = 85 \]

Step 4: Write a mathematical equation.

Step 5: Solve for \( x \).

\[ x = 85 \]

Step 6: Interpret your results.

The first odd integer is \( x \): \( 85 \)
The second odd integer is \( x + 2 \):
\( 85 + 2 = 87 \)

Answer: The integers are 85 and 87.

Skill Practice

2. Four times the sum of three consecutive integers is 264. Find the integers.

3. Applications Involving Percents and Rates

In many real-world applications, percents are used to represent rates.

- The sales tax rate for a certain county is 6%.
- An ice cream machine is discounted 20%.
- A real estate sales broker receives a 4 1/2% commission on sales.
- A savings account earns 7% simple interest.

The following models are used to compute sales tax, commission, and simple interest. In each case the value is found by multiplying the base by the percentage.

\[
\text{Sales tax} = (\text{cost of merchandise})(\text{tax rate})
\]

\[
\text{Commission} = (\text{dollars in sales})(\text{commission rate})
\]

\[
\text{Simple interest} = (\text{principal})(\text{annual interest rate})(\text{time in years})
\]

Answer

2. The integers are 21, 22, and 23.
Chapter 1  Linear Equations and Inequalities in One Variable

**Example 3**  Solving a Percent Application

A woman invests $5000 in an account that earns 5.5% simple interest. If the money is invested for 3 years (yr), how much money is in the account at the end of the 3-yr period?

**Solution:**

Let \( x \) represent the total money in the account.

- \( P = $5000 \) (principal amount invested)
- \( r = 0.0525 \) (interest rate)
- \( t = 3 \) (time in years)

The total amount of money includes principal plus interest.

\[
	ext{(Total money)} = (\text{principal}) + (\text{interest})
\]

**Verbal model**

\[
x = P + Prt
\]

**Mathematical model**

\[
x = $5000 + ($5000)(0.0525)(3)
\]

\[
x = $5000 + $787.50
\]

\[
x = $5787.50
\]

Solve for \( x \).

The total amount of money in the account is $5787.50. Interpret the results.

**Skill Practice**

3. Markos earned $340 in 1 yr on an investment that paid a 4% dividend. Find the amount of money invested.

---

As consumers, we often encounter situations in which merchandise has been marked up or marked down from its original cost. It is important to note that percent increase and percent decrease are based on the original cost. For example, suppose a microwave oven originally priced at $305 is marked down 20%.

The discount is determined by 20% of the original price: \((0.20)($305) = $61.00\).

The new price is $305.00 – $61.00 = $244.00.

**Example 4**  Solving a Percent Increase Application

A college bookstore uses a standard markup of 40% on all books purchased wholesale from the publisher. If the bookstore sells a calculus book for $179.20, what was the original wholesale cost?

---

Answer

3. $8500
Solution:
Let \( x \) = original wholesale cost.

The selling price of the book is based on the original cost of the book plus the bookstore’s markup.

\[
\text{(Selling price)} = (\text{original price}) + (\text{markup})
\]

\[
\text{(Selling price)} = (\text{original price}) + (\text{original price} \times \text{markup rate})
\]

\[
179.20 = x + (0.40)x
\]

\[
179.20 = 1.40x
\]

\[
\frac{179.20}{1.40} = x
\]

\[
x = 128
\]

The original wholesale cost of the textbook was $128.00.

Interpret the results.

Skill Practice

4. An online bookstore gives a 20% discount on paperback books. Find the original price of a book that has a selling price of $5.28 after the discount.

4. Applications Involving Principal and Interest

Example 5 Solving an Investment Growth Application

Miguel had $10,000 to invest in two different mutual funds. One was a relatively safe bond fund that averaged 4% return on his investment at the end of 1 yr. The other fund was a riskier stock fund that averaged 7% return in 1 yr. If at the end of the year Miguel’s portfolio grew to $10,625 ($625 above his $10,000 investment), how much money did Miguel invest in each fund?

Solution:
This type of word problem is sometimes categorized as a mixture problem. Miguel is “mixing” his money between two different investments. We have to determine how the money was divided to earn $625.

The information in this problem can be organized in a chart. (*Note: There are two sources of money: the amount invested and the amount earned.*)

<table>
<thead>
<tr>
<th>Amount Invested ($)</th>
<th>4% Bond Fund</th>
<th>7% Stock Fund</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>((10,000 - x))</td>
<td>10,000</td>
<td></td>
</tr>
<tr>
<td>Amount Earned ($)</td>
<td>0.04( x )</td>
<td>0.07((10,000 - x))</td>
<td>625</td>
</tr>
</tbody>
</table>

Because the amount of principal is unknown for both accounts, we can let \( x \) represent the amount invested in the bond fund. If Miguel spends \( x \) dollars in the bond fund, then he has \((10,000 - x)\) left over to spend in the stock fund. The return for each fund is found by multiplying the principal and the percent growth rate.

Answer

4. $6.60
To establish a mathematical model, we know that the total return ($625) must equal the earnings from the bond fund plus the earnings from the stock fund:

\[
\text{(Earnings from bond fund)} + \text{(earnings from stock fund)} = \text{(total earnings)}
\]

\[
0.04x + 0.07(10,000 - x) = 625
\]

Mathematical model

\[
4x + 7(10,000 - x) = 62,500
\]

Multiply by 100 to clear decimals.

\[
4x + 70,000 - 7x = 62,500
\]

Combine like terms.

\[
-3x + 70,000 = 62,500
\]

Subtract 70,000 from both sides.

\[
-3x = -7500
\]

\[
\frac{-3x}{-3} = \frac{-7500}{-3}
\]

\[
x = 2500
\]

Solve for \(x\) and interpret the results.

The amount invested in the bond fund is $2500.
The amount invested in the stock fund is $10,000 \(- x\), or $7500.

Skill Practice

5. Jonathan borrowed $4000 in two loans. One loan charged 7% interest, and the other charged 1.5% interest. After 1 yr, Jonathan paid $225 in interest. Find the amount borrowed in each loan.

5. Applications Involving Mixtures

Example 6  Solving a Mixture Application

How many liters (L) of a 40% antifreeze solution must be added to 4 L of a 10% antifreeze solution to produce a 35% antifreeze solution?

Solution:

The given information is illustrated in Figure 1-1.
The information can also be organized in a table.

<table>
<thead>
<tr>
<th></th>
<th>40% Antifreeze</th>
<th>10% Antifreeze</th>
<th>Final Solution: 35% Antifreeze</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of liters of solution</td>
<td>$x$</td>
<td>4</td>
<td>$(4 + x)$</td>
</tr>
<tr>
<td>Number of liters of pure antifreeze</td>
<td>$0.40x$</td>
<td>$0.10(4)$</td>
<td>$0.35(4 + x)$</td>
</tr>
</tbody>
</table>

Notice that an algebraic equation is obtained from the second row of the table relating the number of liters of pure antifreeze in each container.

\[
\frac{\text{Pure antifreeze from solution 1}}{0.40x} + \frac{\text{pure antifreeze from solution 2}}{0.10(4)} = \frac{\text{pure antifreeze in the final solution}}{0.35(4 + x)}
\]

\[
0.40x + 0.10(4) = 0.35(x + 4)
\]

Mathematical equation

\[
0.4x + 0.4 = 0.35x + 1.4
\]

Apply the distributive property.

\[
0.4x - 0.35x + 0.4 = 0.35x - 0.35x + 1.4
\]

Subtract 0.35x from both sides.

\[
0.05x + 0.4 = 1.4
\]

\[
0.05x + 0.4 - 0.4 = 1.4 - 0.4
\]

Subtract 0.4 from both sides.

\[
0.05x = 1.0
\]

\[
\frac{0.05x}{0.05} = \frac{1.0}{0.05}
\]

Divide both sides by 0.05.

\[
x = 20
\]

Therefore, 20 L of a 40% antifreeze solution is needed.

**Skill Practice**

6. Find the number of ounces (oz) of 30% alcohol solution that must be mixed with 10 oz of a 70% solution to obtain a solution that is 40% alcohol.

**6. Applications Involving Distance, Rate, and Time**

The fundamental relationship among the variables distance, rate, and time is given by

\[
\text{Distance} = (\text{rate})(\text{time}) \quad \text{or} \quad d = rt
\]

For example, a motorist traveling 65 mph (miles per hour) for 3 hr (hours) will travel a distance of

\[
d = \left(\frac{65 \text{ mi}}{\text{hr}}\right)(3 \text{ hr}) = 195 \text{ mi}
\]

**Answer**

6. 30 oz of the 30% solution is needed.
Example 7  Solving a Distance, Rate, Time Application

A hiker can hike 1 mph faster downhill to Moose Lake than she can hike uphill back to the campsite. If it takes her 3 hr to hike to the lake and 4.5 hr to hike back, what is her speed hiking back to the campsite?

Solution:
The information given in the problem can be organized in a table.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Rate (mph)</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip to the lake</td>
<td>x + 1</td>
<td>3</td>
</tr>
<tr>
<td>Return trip</td>
<td>x</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Column 2: Let the rate of the return trip be represented by x. Then the trip to the lake is 1 mph faster and can be represented by x + 1.

Column 3: The times hiking to and from the lake are given in the problem.

Column 1: To express the distance, we use the relationship $d = rt$. That is, multiply the quantities in the second and third columns.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Rate (mph)</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip to the lake</td>
<td>3(x + 1)</td>
<td>3</td>
</tr>
<tr>
<td>Return trip</td>
<td>4.5x</td>
<td>4.5</td>
</tr>
</tbody>
</table>

To create a mathematical model, note that the distances to and from the lake are equal. Therefore,

$$3(x + 1) = 4.5x$$

$3x + 3 = 4.5x$

$3x - 3x + 3 = 4.5x - 3x$

$3 = 1.5x$

$$\frac{3}{1.5} = \frac{1.5x}{1.5}$$

$2 = x$

Solve for x.

The hiker’s speed on the return trip to the campsite is 2 mph.

Skill Practice

7. During a bad rainstorm, Jody drove 15 mph slower on a trip to her mother’s house than she normally would when the weather is clear. If a trip to her mother’s house takes 3.75 hr in clear weather and 5 hr in a bad storm, what is her normal driving speed during clear weather?
Study Skills Exercises

1. After doing a section of homework, check the odd-numbered answers in the back of the text. Choose a method to identify the exercises that gave you trouble (i.e., circle the number or put a star by the number). List some reasons why it is important to label these problems.

2. Define the key terms.
   a. Sum
   b. Difference
   c. Product
   d. Quotient
   e. Sales tax
   f. Commission
   g. Simple interest

Review Exercises

For Exercises 3–8, solve the equations.

3. \( 7a - 2 = 11 \)
4. \( 2x + 6 = -15 \)
5. \( 4(x - 3) + 7 = 19 \)
6. \( -3(y - 5) + 4 = 1 \)
7. \( \frac{3}{8}p + \frac{3}{4} = p - \frac{3}{2} \)
8. \( \frac{1}{4} - 2x = 5 \)

For the remaining exercises, follow the steps outlined in the Problem-Solving Flowchart found on page 57.

Concept 1: Introduction to Problem Solving

9. If \( x \) represents a number, write an expression for 5 more than the number.

10. If \( n \) represents a number, write an expression for 10 less than the number.

11. If \( t \) represents a number, write an expression for 7 less than twice the number.

12. If \( y \) represents a number, write an expression for 4 more than 3 times the number.

13. The larger of two numbers is 3 more than twice the smaller. The difference of the larger number and the smaller number is 8. Find the numbers. (See Example 1.)

14. One number is 3 less than another. Their sum is 15. Find the numbers.

15. The sum of 3 times a number and 2 is the same as the difference of the number and 4. Find the number.

16. Twice the sum of a number and 3 is the same as 1 subtracted from the number. Find the number.

17. The sum of two integers is 30. Ten times one integer is 5 times the other integer. Find the integers. (Hint: If one number is \( x \), then the other number is \( 30 - x \).)

18. The sum of two integers is 10. Three times one integer is 3 less than 8 times the other integer. Find the integers. (Hint: If one number is \( x \), then the other number is \( 10 - x \).)

Concept 2: Applications Involving Consecutive Integers

19. The sum of two consecutive page numbers in a book is 223. Find the page numbers. (See Example 2.)

20. The sum of the numbers on two consecutive raffle tickets is 808,455. Find the numbers on the tickets.
Chapter 1  Linear Equations and Inequalities in One Variable

21. The sum of two consecutive odd integers is −148. Find the two integers.

22. The sum of three consecutive integers is −57. Find the integers.

23. Three times the smaller of two consecutive even integers is the same as −146 minus 4 times the larger integer. Find the integers.

24. Four times the smaller of two consecutive odd integers is the same as 73 less than 5 times the larger. Find the integers.

25. Two times the sum of three consecutive odd integers is the same as 23 more than 5 times the largest integer. Find the integers.

26. Five times the smallest of three consecutive even integers is 10 more than twice the largest. Find the integers.

Concept 3: Applications Involving Percents and Rates

27. Belle had the choice of taking out a 4-yr car loan at 8.5% simple interest or a 5-yr car loan at 7.75% simple interest. If she borrows $15,000, how much interest would she pay for each loan? Which option will require less interest? (See Example 3.)

28. Robert can take out a 3-yr loan at 8% simple interest or a 2-yr loan at 8 1/4% simple interest. If he borrows $7000, how much interest will he pay for each loan? Which option will require less interest?

29. An account executive earns $600 per month plus a 3% commission on sales. The executive’s goal is to earn $2400 this month. How much must she sell to achieve this goal?

30. A salesperson earns $50 a day plus 12% commission on sales over $200. If her daily earnings are $76.88, how much money in merchandise did she sell?

31. J.W. is an artist and sells his pottery each year at a local Renaissance Festival. He keeps track of his sales and the 8.05% sales tax he collects by making notations in a ledger. Every evening he checks his records by counting the total money in his cash drawer. After a day of selling pottery, the cash totaled $1293.38. How much is from the sale of merchandise and how much is sales tax?

32. Wayne County has a sales tax rate of 7%. How much does Mike’s used Honda Civic cost before tax if the total cost of the car plus tax is $13,888.60?

33. The price of a swimsuit after a 20% markup is $43.08. What was the price before the markup? (See Example 4.)

34. The price of a used textbook after a 35% markdown is $29.25. What was the original price?

35. For a recent year, 1800 medical degrees were awarded to women. This represents a 5.5% increase over the number awarded the previous year. How many women were awarded a medical degree the previous year?

36. For a recent year, Americans spent approximately $69 billion on weddings. This represents a 50% increase from the amount spent in 2001. What amount did Americans spend on weddings in 2001?
Section 1.2 Applications of Linear Equations in One Variable

Concept 4: Applications Involving Principal and Interest

37. Tony has a total of $12,500 in two accounts. One account pays 2% simple interest per year and the other pays 5% simple interest. If he earned $370 in interest in the first year, how much did he invest in each account? (See Example 5.)

38. Lillian had $15,000 invested in two accounts, one paying 9% simple interest and one paying 10% simple interest. How much was invested in each account if the interest after 1 yr is $1432?

39. Jason borrowed $18,000 in two loans. One loan charged 11% simple interest and the other charged 6% simple interest. After 1 yr, Jason paid a total of $1380. Find the amount borrowed in each loan.

40. Amanda borrowed $6000 from two sources: her parents and a credit union. Her parents charged 3% simple interest and the credit union charged 8% simple interest. If after 1 yr, Amanda paid $255 in interest, how much did she borrow from her parents, and how much did she borrow from the credit union?

41. Donna invested money in two accounts: one paying 4% simple interest and the other paying 3% simple interest. She invested $4000 more in the 4% account than in the 3% account. If she received $720 in interest at the end of 1 yr how much did she invest in each account?

42. Mr. Hall had some money in his bank earning 4.5% simple interest. He had $5000 more deposited in a credit union earning 6% simple interest. If his total interest for 1 yr was $1140, how much did he deposit in each account?

43. Ms. Riley deposited some money in an account paying 5% simple interest and twice that amount in an account paying 6% simple interest. If the total interest from the two accounts is $765 for 1 yr, how much was deposited into each account?

44. Sienna put some money in a certificate of deposit earning 4.2% simple interest. She deposited twice that amount in a money market account paying 4% simple interest. After 1 yr her total interest was $488. How much did Sienna deposit in her money market account?

Concept 5: Applications Involving Mixtures

45. Ahmed mixes two plant fertilizers. How much fertilizer with 15% nitrogen should be mixed with 2 oz of fertilizer with 10% nitrogen to produce a fertilizer that is 14% nitrogen? (See Example 6.)

46. How much 8% saline solution should Kent mix with 80 cc (cubic centimeters) of an 18% saline solution to produce a 12% saline solution?

47. Jacque has 3 L of a 50% antifreeze mixture. How much 75% mixture should be added to get a mixture that is 60% antifreeze?

48. One fruit punch has 40% fruit juice and another is 70% fruit juice. How much of the 40% punch should be mixed with 10 gal of the 70% punch to create a fruit punch that is 45% fruit juice?

49. How many liters of an 18% alcohol solution must be added to a 10% alcohol solution to get 20 L of a 15% alcohol solution?

50. How many milliliters of a 2.5% bleach solution must be mixed with 10 gal of the 70% punch to produce 600 mL of a 5% bleach solution?

51. Ronald has a 12% solution of the fertilizer Super Grow. How much pure Super Grow should he add to the mixture to get 32 oz of a 17.5% concentration?

52. How many ounces of water must be added to 20 oz of an 8% salt solution to make a 2% salt solution?

53. Two different teas are mixed to make a blend that will be sold at a fair. Black tea sells for $2.20 per pound and green tea sells for $3.00 per pound. How much of each should be used to obtain 4 lb of a blend selling for $2.50?

54. A nut mixture consists of almonds and cashews. Almonds are $4.98 per pound, and cashews are $6.98 per pound. How many pounds of each type of nut should be mixed to produce 16 lb selling for $5.73 per pound?
Concept 6: Applications Involving Distance, Rate, and Time

55. A Piper Cub airplane has an average speed that is 30 mph faster than a Cessna 150 airplane. It takes the Cessna 5 hr to fly from Fort Lauderdale to Atlanta, while it takes the Piper Cub only 4 hr to make the same trip. What is the average speed of each plane? (See Example 7.)

56. A woman can hike 1 mph faster down a trail to Archuletta Lake than she can on the return trip uphill. It takes her 3 hr to get to the lake and 6 hr to return. What is her speed hiking down to the lake?

57. Two cars are 192 mi apart and travel toward each other on the same road. They meet in 2 hr. One car travels 4 mph faster than the other. What is the average speed of each car?

58. Two cars are 190 mi apart and travel toward each other along the same road. They meet in 2 hr. One car travels 5 mph slower than the other car. What is the average speed of each car?

59. Two boats traveling the same direction leave a harbor at noon. After 3 hr they are 60 mi apart. If one boat travels twice as fast as the other, find the average rate of each boat.

60. Two canoes travel down a river, starting at 9:00. One canoe travels twice as fast as the other. After 3.5 hr, the canoes are 5.25 mi apart. Find the average rate of each canoe.

Section 1.3 Applications to Geometry and Literal Equations

1. Applications Involving Geometry

Some word problems involve the use of geometric formulas such as those listed in the inside back cover of this text.

Example 1 Solving an Application Involving Perimeter

The length of a rectangular corral is 2 ft more than 3 times the width. The corral is situated such that one of its shorter sides is adjacent to a barn and does not require fencing. If the total amount of fencing is 774 ft, then find the dimensions of the corral.

Solution:

Read the problem and draw a sketch (Figure 1-2).

Let x represent the width. Label variables.

Let $3x + 2$ represent the length.
To create a verbal model, we might consider using the formula for the perimeter of a rectangle. However, the formula \( P = 2l + 2w \) incorporates all four sides of the rectangle. The formula must be modified to include only one factor of the width.

\[
\left( \begin{array}{c}
\text{Distance around three sides} \\
774
\end{array} \right) = \left( \begin{array}{c}
2 \times \text{the length} \\
2(3x + 2)
\end{array} \right) + \left( \begin{array}{c}
1 \times \text{the width} \\
x
\end{array} \right)
\]

Verbal model

Mathematical model

\( 774 = 2(3x + 2) + x \)

Solve for \( x \).

\( 774 = 6x + 4 + x \)

Apply the distributive property.

\( 774 = 7x + 4 \)

Combine like terms.

\( 770 = 7x \)

Subtract 4 from both sides.

\( 110 = x \)

Divide by 7 on both sides.

\( x = 110 \)

Because \( x \) represents the width, the width of the corral is 110 ft. The length is given by

\[
3x + 2 \quad \text{or} \quad 3(110) + 2 = 332
\]

Interpret the results.

The width of the corral is 110 ft, and the length is 332 ft.

**Skill Practice**

1. The length of Karen’s living room is 2 ft longer than the width. The perimeter is 80 ft. Find the length and width.

**Avoiding Mistakes**

To check the answer to Example 1, verify that the three sides add to 774 ft.

\[
110 \text{ ft} + 332 \text{ ft} + 332 \text{ ft} = 774 \text{ ft} \quad \checkmark
\]

Recall some important facts involving angles.

- Two angles are complementary if the sum of their measures is 90°.
- Two angles are supplementary if the sum of their measures is 180°.
- The sum of the measures of the angles within a triangle is 180°.

**Example 2** Solving an Application Involving Angles

Two angles are complementary. One angle measures 10° less than 4 times the other angle. Find the measure of each angle (Figure 1-3).

**Solution:**

Let \( x \) represent the measure of one angle.

Let \( 4x - 10 \) represent the measure of the other angle.

Recall that two angles are complementary if the sum of their measures is 90°. Therefore, a verbal model is

\[
(\text{One angle}) + (\text{the complement of the angle}) = 90°
\]

Verbal model

Mathematical equation

\[ x + (4x - 10) = 90 \]

\[ 5x - 10 = 90 \]

Solve for \( x \).

**Answer**

1. The length is 21 ft, and the width is 19 ft.
5x = 100
x = 20

If x = 20, then 4x - 10 = 4(20) - 10 = 70. The two angles are 20° and 70°.

**Skill Practice**

2. Two angles are supplementary, and the measure of one is 16° less than 3 times the other. Find their measures.

2. **Literal Equations**

Literal equations are equations that contain several variables. A formula is a literal equation with a specific application. For example, the perimeter of a rectangle can be found by the formula \( P = 2l + 2w \). In this equation, \( P \) is expressed in terms of \( l \) and \( w \). However, in science and other branches of applied mathematics, formulas may be more useful in alternative forms.

For example, the formula \( P = 2l + 2w \) can be manipulated to solve for either \( l \) or \( w \):

- **Solve for \( l \)**
  
  \[
  P = 2l + 2w \\
  P - 2w = 2l \\
  \frac{P - 2w}{2} = l
  \]

- **Solve for \( w \)**
  
  \[
  P = 2l + 2w \\
  P - 2l = 2w \\
  \frac{P - 2l}{2} = w
  \]

To solve a literal equation for a specified variable, use the addition, subtraction, multiplication, and division properties of equality.

**Example 3** **Applying a Literal Equation**

Buckingham Fountain is one of Chicago’s most familiar landmarks. With 133 jets spraying a total of 14,000 gal of water per minute, Buckingham Fountain is one of the world’s largest fountains. The circumference of the fountain is approximately 880 ft.

a. The circumference of a circle is given by \( C = 2\pi r \). Solve the equation for \( r \).

b. Use the equation from part (a) to find the radius and diameter of the fountain.

Use 3.14 for \( \pi \) and round to the nearest foot.

**Solution:**

a. \( C = 2\pi r \)

\[
\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \\
\frac{C}{2\pi} = r
\]

\( r = \frac{C}{2\pi} \)

**Answer**

2. 49° and 131°
b. \( r \approx \frac{880}{2(3.14)} \) Substitute 880 ft for \( C \) and 3.14 for \( \pi \).

\[ r \approx 140 \text{ ft} \]

The radius is approximately 140 ft. The diameter is twice the radius \( (d = 2r) \). Therefore, the diameter is 280 ft.

**Skill Practice**  The formula to compute the surface area \( S \) of a sphere is given by \( S = 4\pi r^2 \).

3. Solve the equation for \( \pi \).
4. A sphere has a surface area of 113 in.\(^2\) and a radius of 3 in. Use the formula found in part (a) to approximate \( \pi \). Round to two decimal places.

### Example 4  Solving a Literal Equation

The formula to find the area of a trapezoid is given by \( A = \frac{1}{2}(b_1 + b_2)h \), where \( b_1 \) and \( b_2 \) are the lengths of the parallel sides and \( h \) is the height. (See Figure 1-4.)

Solve this formula for \( b_1 \).

**Solution:**

\[ A = \frac{1}{2}(b_1 + b_2)h \]

The goal is to isolate \( b_1 \).

\[ 2A = 2 \cdot \frac{1}{2}(b_1 + b_2)h \]

Multiply by 2 to clear fractions.

\[ 2A = (b_1 + b_2)h \]

Apply the distributive property.

\[ 2A = b_1h + b_2h \]

\[ 2A - b_2h = b_1h \]

Subtract \( b_2h \) from both sides.

\[ \frac{2A - b_2h}{h} = b_1 \]

Divide by \( h \).

**Skill Practice**

5. The formula for the volume of a right circular cylinder is \( V = \pi r^2h \). Solve for \( h \).

### TIP:  When solving a literal equation for a specified variable, there is sometimes more than one way to express your final answer. This flexibility often presents difficulty for students. Students may leave their answer in one form, but the answer given in the text looks different. Yet both forms may be correct. To know if your answer is equivalent to the form given in the text you must try to manipulate it to look like the answer in the book, a process called form fitting.

The literal equation from Example 4 can be written in several different forms. The quantity \( (2A - b_2h)/h \) can be split into two fractions.

**Answers**

3. \( \pi = \frac{S}{4r^2} \)  
4. 3.14  
5. \( h = \frac{V}{\pi r^2} \)
Chapter 1  Linear Equations and Inequalities in One Variable

Example 5  Solving a Literal Equation

Given $-2x + 3y = 5$, solve for $y$.

Solution:

$$-2x + 3y = 5$$

Add $2x$ to both sides.

$$3y = 2x + 5$$

Divide by 3 on both sides.

$$\frac{3y}{3} = \frac{2x + 5}{3}$$

$$y = \frac{2x + 5}{3}$$

Skill Practice  Solve for $y$.

6. $5x + 2y = 11$

Sometimes the variable we want to isolate may appear in more than one term in a literal equation. In such a case, isolate all terms with that variable on one side of the equation. Then apply the distributive property as demonstrated in Example 6.

Example 6  Solving a Literal Equation

Solve the equation for $x$. $ax - 3 = cx + 7$

Solution:

$$ax - 3 = cx + 7$$

Collect the terms containing $x$ on one side of the equation. Collect the remaining terms on the other side.

The variable $x$ appears twice in the equation. To isolate $x$, we want $x$ to appear in only one term. To accomplish this, we apply the distributive property in reverse.

$$x(a - c) = 10$$

Apply the distributive property. The variable $x$ now appears one time in the equation.

$$\frac{x(a - c)}{(a - c)} = \frac{10}{(a - c)}$$

$$x = \frac{10}{a - c}$$

Skill Practice  Solve for $t$.

7. $mt + 4 = nt + 9$

Answers

6. $y = \frac{11 - 5x}{2}$ or $y = \frac{5}{2}x + \frac{11}{2}$

7. $t = \frac{5}{m - n}$
**Section 1.3  Applications to Geometry and Literal Equations**

### Practice Exercises

**Study Skills Exercise**

1. In your next math class, take notes by drawing a vertical line about three-fourths of the way across the paper, as shown. On the left side, write down what your instructor puts on the board or overhead. On the right side, make your own comments about important words, procedures, or questions that you have.

**Review Exercises**

For Exercises 2–6, solve the equations.

2. \[7 + 5x - (2x - 6) = 6(x + 1) + 21\]

3. \[\frac{3}{5}y - 3 + 2y = 5\]

4. \[3[z - (2 - 3z) - 4] = z - 7\]

5. \[2a - 4 + 8a = 7a - 8 + 3a\]

6. \[3(t + 6) + t + 2 = 5(t + 4) - t\]

**Concept 1: Applications Involving Geometry**

For Exercises 7–18, use the geometry formulas listed in the inside back cover of the text.

7. A volleyball court is twice as long as it is wide. If the perimeter is 177 ft, find the dimensions of the court. (See Example 1.)

8. The length of a rectangular picture frame is 4 in. less than twice the width. The perimeter is 112 in. Find the length and the width.

9. The lengths of the sides of a triangle are given by three consecutive even integers. The perimeter is 24 m. What is the length of each side?

10. A triangular garden has sides that can be represented by three consecutive integers. If the perimeter of the garden is 15 ft, what are the lengths of the sides?

11. Raoul would like to build a rectangular dog run in the rear of his backyard, away from the house. The width of the yard is \(12\frac{1}{2}\) yd, and Raoul wants an area of 100 yd\(^2\) for his dog.
   a. Find the dimensions of the dog run.
   b. How much fencing would Raoul need to enclose the dog run?

12. Joanne wants to plant a flower garden in her backyard in the shape of a trapezoid, adjacent to her house (see the figure). She also wants a front yard garden in the same shape, but with sides one-half as long. What should the dimensions be for each garden if Joanne has only a total of 60 ft of fencing?
13. George built a rectangular pen for his rabbit such that the length is 7 ft less than twice the width. If the perimeter is 40 ft, what are the dimensions of the pen?

14. Antoine wants to put edging in the form of a square around a tree in his front yard. He has enough money to buy 18 ft of edging. Find the dimensions of the square that will use all the edging.

15. The measures of two angles in a triangle are equal. The third angle measures 2 times the sum of the equal angles. Find the measures of the three angles.

16. The smallest angle in a triangle is one-half the size of the largest. The middle angle measures 25° less than the largest. Find the measures of the three angles.

17. Two angles are complementary. One angle is 5 times as large as the other angle. Find the measure of each angle. (See Example 2.)

18. Two angles are supplementary. One angle measures 12° less than 3 times the other. Find the measure of each angle.

In Exercises 19–26, solve for \( x \), and then find the measure of each angle.

19. \[ (7x - 1)^\circ \quad (2x + 1)^\circ \]

20. \[ (10x + 36)^\circ \quad [2(x + 15)]^\circ \]

21. \[ (2x + 5)^\circ \quad (x + 2.5)^\circ \]

22. \[ (3x - 3)^\circ \quad [3(5x + 1)]^\circ \]

23. \[ (2x)^\circ \quad (5x + 1)^\circ \quad (x + 35)^\circ \]

24. \[ (10x)^\circ \quad (x - 2)^\circ \quad (20x - 4)^\circ \]

25. \[ (2x - 4)^\circ \quad [3(x - 7)]^\circ \]

26. \[ (x + 2)^\circ \quad [4(x - 8)]^\circ \]
For Exercises 31–48, solve for the indicated variable. (See Example 4.)
31. \(A = lw\) for \(l\)  
32. \(C_1 = \frac{2}{3}R\) for \(R\)  
33. \(I = Prt\) for \(P\)  
34. \(a + b + c = P\) for \(b\)  
35. \(W = K_2 - K_1\) for \(K_1\)  
36. \(y = mx + b\) for \(x\)  
37. \(F = \frac{9}{5}C + 32\) for \(C\)  
38. \(C = \frac{5}{9}(F - 32)\) for \(F\)  
39. \(K = \frac{1}{2}mv^2\) for \(v^2\)  
40. \(I = Prt\) for \(r\)  
41. \(v = v_0 + at\) for \(a\)  
42. \(a^2 + b^2 = c^2\) for \(b^2\)  
43. \(w = p(v_2 - v_1)\) for \(v_2\)  
44. \(A = lw\) for \(w\)  
45. \(ax + by = c\) for \(y\)  
46. \(P = 2L + 2W\) for \(L\)  
47. \(V = \frac{1}{3}Bh\) for \(B\)  
48. \(V = \frac{1}{3} \pi r^2h\) for \(h\)

For Exercises 49–60, express each equation in the form \(y = mx + b\) by solving for \(y\). (See Example 5.)
49. \(3x + y = 6\)  
50. \(x + y = -4\)  
51. \(5x - 4y = 20\)  
52. \(-4x - 5y = 25\)  
53. \(-6x - 2y = 13\)  
54. \(5x - 7y = 15\)  
55. \(3x - 3y = 6\)  
56. \(2x - 2y = 8\)  
57. \(9x + \frac{4}{3}y = 5\)  
58. \(4x - \frac{1}{3}y = 5\)  
59. \(-x + \frac{2}{3}y = 0\)  
60. \(x - \frac{1}{4}y = 0\)
In statistics, the z-score formula $z = \frac{x - \mu}{\sigma}$ is used in studying probability. Use this formula for Exercises 61–62.

61. a. Solve $z = \frac{x - \mu}{\sigma}$ for $x$.
   b. Find $x$ when $z = 2.5$, $\mu = 100$, and $\sigma = 12$.

62. a. Solve $z = \frac{x - \mu}{\sigma}$ for $\sigma$.
   b. Find $\sigma$ when $x = 150$, $z = 2.5$, and $\mu = 110$.

63. Which expressions are equivalent to $\frac{-5}{x - 3}$?
   a. $\frac{5}{x - 3}$
   b. $\frac{5}{3 - x}$
   c. $\frac{-5}{x + 3}$

64. Which expressions are equivalent to $\frac{z - 1}{2}$?
   a. $\frac{1 - z}{2}$
   b. $\frac{z - 1}{2}$
   c. $\frac{-z + 1}{2}$

65. Which expressions are equivalent to $\frac{-x - 7}{y}$?
   a. $\frac{x + 7}{y}$
   b. $\frac{x + 7}{-y}$
   c. $\frac{-x - 7}{-y}$

66. Which expressions are equivalent to $\frac{-3w}{x - y}$?
   a. $\frac{3w}{x - y}$
   b. $\frac{3w}{x + y}$
   c. $\frac{-3w}{x + y}$

For Exercises 67–75, solve for the indicated variable. (See Example 6.)

67. $6t - rt = 12$ for $t$
68. $5 = 4a + ca$ for $a$
69. $ax + 5 = 6x + 3$ for $x$

70. $cx - 4 = dx + 9$ for $x$
71. $A = P + Prt$ for $P$
72. $A = P + Prt$ for $r$

73. $T = mg - mf$ for $m$
74. $T = mg - mf$ for $f$
75. $ax + by = cx + z$ for $x$

---

**Section 1.4 Linear Inequalities in One Variable**

**Concepts**

1. Solving Linear Inequalities
2. Applications of Inequalities

1. **Solving Linear Inequalities**

In Sections 1.1–1.3, we learned how to solve linear equations and their applications. In this section, we will learn the process of solving linear inequalities. A **linear inequality** in one variable, $x$, is defined as any relationship of the form: $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, or $ax + b \geq 0$, where $a \neq 0$.

The solution to the equation $x = 3$ can be graphed as a single point on the number line.

Now consider the inequality $x \leq 3$. The solution set to an inequality is the set of real numbers that makes the inequality a true statement. In this case, the solution set is all real numbers less than or equal to 3. Because the solution set has an infinite number of values, the values cannot be listed. Instead, we can graph the solution set or represent the set in interval notation or in set-builder notation. A complete discussion of set-builder notation and interval notation is given in Section R.2.

Graph |
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<td>$-4$</td>
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<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Set-Builder Notation</th>
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<tr>
<td>$(-\infty, 3]$</td>
<td>{ $x$</td>
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</table>
The addition and subtraction properties of equality indicate that a value added to or subtracted from both sides of an equation results in an equivalent equation. The same is true for inequalities.

**PROPERTY Addition and Subtraction Properties of Inequality**

Let $a$, $b$, and $c$ represent real numbers.

*Addition property of inequality:* If $a < b$
then $a + c < b + c$

*Subtraction property of inequality:* If $a < b$
then $a - c < b - c$

*These properties may also be stated for and $a > b$, $a < b$, and $a \geq b$.

**Example 1 Solving a Linear Inequality**

Solve the inequality. Graph the solution and write the solution set in interval notation.

$$3x - 7 > 2(x - 4) - 1$$

**Solution:**

$$3x - 7 > 2(x - 4) - 1$$
$$3x - 7 > 2x - 8 - 1$$
$$3x - 7 > 2x - 9$$

Subtract $2x$ from both sides.

$$x - 7 > -9$$

Add 7 to both sides.

$$x > -2$$

**Graph**

-4 -3 -2 -1 0 1 2 3 4

**Interval Notation**

$(-2, \infty)$

**Skill Practice** Solve the inequality. Graph the solution and write the solution set in interval notation.

1. $4(2x - 1) > 7x + 1$

Multiplying both sides of an equation by the same quantity results in an equivalent equation. However, the same is not always true for an inequality. If you multiply or divide an inequality by a negative quantity, the direction of the inequality symbol must be reversed.

For example, consider multiplying or dividing the inequality $4 < 5$ by $-1$.

Multiply/divide by $-1$:

$$4 < 5$$
$$-4 > -5$$

The number 4 lies to the left of 5 on the number line. However, $-4$ lies to the right of $-5$. Changing the signs of two numbers changes their relative position on the number line. This is stated formally in the multiplication and division properties of inequality.
PROPERTY Multiplication and Division Properties of Inequality

Let \( a, b, \) and \( c \) represent real numbers.

*If \( c \) is positive and \( a < b \), then \( ac < bc \) and \( \frac{a}{c} < \frac{b}{c} \)

*If \( c \) is negative and \( a < b \), then \( ac > bc \) and \( \frac{a}{c} > \frac{b}{c} \)

The second statement indicates that if both sides of an inequality are multiplied or divided by a negative quantity, the inequality sign must be reversed.

*These properties may also be stated for \( a \leq b, a > b, \) and \( a \geq b \).

Example 2 Solving a Linear Inequality

Solve the inequality. Graph the solution and write the solution set in interval notation.

\[-2x - 5 < 2\]

Solution:

\[-2x - 5 < 2\]

\[-2x - 5 + 5 < 2 + 5\] Add 5 to both sides.

\[-2x < 7\]

\[-\frac{2x}{-2} > \frac{7}{-2}\] Divide by \(-2\) (reverse the inequality sign).

\[x > -\frac{7}{2}\] or \[x > -3.5\]

Graph Interval Notation

\[\left(-\frac{7}{2}, \infty\right)\]

TIP: The inequality \(-2x - 5 < 2\) could have been solved by isolating \(x\) on the right-hand side of the inequality. This creates a positive coefficient on the \(x\) term and eliminates the need to divide by a negative number.

\[-2x - 5 < 2\]

\[-5 < 2x + 2\] Add 2 to both sides.

\[-7 < 2x\] Subtract 2 from both sides.

\[-\frac{7}{2} < 2x\] Divide by 2 (because 2 is positive, do not reverse the inequality sign).

\[-\frac{7}{2} < x\] (Note that the inequality \(-\frac{7}{2} < x\) is equivalent to \(x > -\frac{7}{2}\))

Skill Practice Solve the inequality. Graph the solution set and write the solution in interval notation.

2. \(-4x - 12 \geq 20\)
Section 1.4  Linear Inequalities in One Variable

### Example 3  Solving a Linear Inequality

Solve the inequality. Graph the solution and write the solution set in interval notation.

\[-6(x - 3) \geq 2 - 2(x - 8)\]

**Solution:**

\[-6(x - 3) \geq 2 - 2(x - 8)\]

\[-6x + 18 \geq 2 - 2x + 16\]  
Apply the distributive property.

\[-6x + 18 \geq 18 - 2x\]

\[-6x + 2x + 18 \geq 18 - 2x + 2x\]

Combine like terms.

\[-4x + 18 \geq 18\]

\[-4x + 18 - 18 \geq 18 - 18\]

Add 2x to both sides.

\[-4x \geq 0\]

Subtract 18 from both sides.

\[-4x \leq 0\]

Divide by -4 (reverse the inequality sign).

\[x \leq 0\]

**Graph**

**Interval Notation**

\[(-\infty, 0]\]

### Skill Practice

Solve the inequality. Graph the solution and write the solution set in interval notation.

3. \(5(3x + 1) < 4(5x - 5)\)

### Example 4  Solving a Linear Inequality

Solve the inequality \(\frac{-5x + 2}{-3} > x + 2\). Graph the solution and write the solution set in interval notation.

**Solution:**

\[\frac{-5x + 2}{-3} > x + 2\]

\[\frac{-3(-5x + 2)}{-3} < \frac{-3(x + 2)}{-3}\]  
Multiply by -3 to clear fractions (reverse the inequality sign).

\[-5x + 2 < -3x - 6\]

\[-2x + 2 < -6\]

Add 3x to both sides.

\[-2x < -8\]

Subtract 2 from both sides.

\[-2x > -8\]

Divide by -2 (the inequality sign is reversed again).

\[x > 4\]

Simplify.

**Answer**

3. \((5, \infty)\)
In Example 4, the inequality sign was reversed twice: once for multiplying the inequality by \(- \frac{1}{3}\) and once for dividing by \(-2\). If you are in doubt about whether you have the inequality sign in the correct direction, you can check your final answer by using the test point method. That is, pick a point in the proposed solution set, and verify that it makes the original inequality true. Furthermore, any test point picked outside the solution set should make the original inequality false.

Pick \(x = 0\) as a test point

\[
\frac{-5x + 2}{-3} \geq x + 2
\]

\[
\frac{-5(0) + 2}{-3} \geq (0) + 2
\]

\[
\frac{2}{-3} \geq 2 \quad \text{False}
\]

Pick \(x = 5\) as a test point

\[
\frac{-5x + 2}{-3} > x + 2
\]

\[
\frac{-5(5) + 2}{-3} > (5) + 2
\]

\[
\frac{-23}{-3} > 7
\]

\[
\frac{7}{2} > 7 \quad \text{True}
\]

Because a test point to the right of \(x = 4\) makes the inequality true, we have shaded the correct part of the number line.

2. Applications of Inequalities

**Example 5** Solving a Linear Inequality Application

Beth received grades of 97%, 82%, 89%, and 99% on her first four algebra tests. To earn an A in the course, she needs an average of 90% or more. What scores can she receive on the fifth test to earn an A?

**Solution:**

Let \(x\) represent the score on the fifth test.

The average of the five tests is given by

\[
\frac{97 + 82 + 89 + 99 + x}{5}
\]
To earn an A, we have:

\[
\frac{97 + 82 + 89 + 99 + x}{5} \geq 90
\]

Verbal model

Mathematical model

\[
\frac{367 + x}{5} \geq 90
\]

Simplify the numerator.

Clear fractions.

\[
\frac{5 \left( \frac{367 + x}{5} \right)}{5} \geq 5(90)
\]

Simplify.

\[
x \geq 83
\]

To earn an A, Beth would have to score at least 83% on her fifth test.

**Skill Practice**

5. Jamie is a salesman who works on commission, so his salary varies from month to month. To qualify for an automobile loan, his monthly salary must average at least $2100 for 6 months. His salaries for the past 5 months have been $1800, $2300, $1500, $2200, and $2800. What amount does he need to earn in the last month to qualify for the loan?

\[
x / 8350 = 83
\]

To earn an A, Beth would have to score at least 83% on her fifth test.

**Example 6  Solving a Linear Inequality Application**

The number of registered passenger cars, \( N \) (in millions), in the United States has risen since 1960 according to the equation \( N = 2.5t + 64.4 \), where \( t \) represents the number of years after 1960 (\( t = 0 \) corresponds to 1960, \( t = 1 \) corresponds to 1961, and so on) (Figure 1-5).

![Graph of N vs. t](image)

**Figure 1-5**

Source: U.S. Department of Transportation

For what years was the number of registered passenger cars less than 89.4 million?

**Answer**

5. Jamie’s salary must be at least $2000.
Chapter 1 Linear Equations and Inequalities in One Variable

Solution:
We require \( N < 89.4 \) million.

\[
\begin{align*}
N &< 89.4 \\
2.5t + 64.4 &< 89.4 \\
2.5t &< 25 \\
\frac{2.5t}{2.5} &< \frac{25}{2.5} \\
t &< 10
\end{align*}
\]

Substitute the expression \( 2.5t + 64.4 \) for \( N \).

Subtract 64.4 from both sides.

Divide both sides by 2.5.

Before 1970, the number of registered passenger cars was less than 89.4 million.

Skill Practice
6. The population of Alaska has steadily increased since 1950 according to the equation \( P = 10t + 117 \), where \( t \) represents the number of years after 1950 and \( P \) represents the population in thousands. For what years since 1950 was the population less than 417 thousand people?

Answer
6. The population was less than 417 thousand for \( t < 30 \). This corresponds to the years before 1980.

Section 1.4 Practice Exercises

Study Skills Exercises
1. Look over the notes that you took today. Do you understand what you wrote? If there were any rules, definitions, or formulas, highlight them so that they can be easily found when studying for the test. You may want to begin by highlighting the rule indicating when the direction of an inequality sign must be reversed.

2. Define the key terms.
   a. Linear inequality
   b. Test point method

Review Exercises
For Exercises 3–4, solve the equation.

3. \( 4 + 5(4 - 2x) = -2(x - 1) - 4 \)

4. \( \frac{1}{5}t - \frac{1}{2} - \frac{1}{10}t + \frac{2}{5} = \frac{3}{10}t + \frac{1}{2} \)

5. Solve for \( v \).
   \( d = vt - 16t^2 \)

6. Solve for \( y \).
   \( 5x + 3y + 6 = 0 \)

7. a. The area of a triangle is given by \( A = \frac{1}{2}bh \). Solve for \( h \).
   b. If the area of a triangle is 10 cm\(^2\) and the base is 3 cm, find the height.

8. Five more than 3 times a number is 6 less than twice the number. Find the number.
Concept 1: Solving Linear Inequalities

For Exercises 9–46, solve the inequalities. Graph the solution and write the solution set in interval notation. Check each answer by using the test point method. (See Examples 1–4.)

9. $2y + 6 \leq 4$
10. $3y + 11 > 5$
11. $-2x - 5 \leq -25$

12. $-4z - 2 > -22$
13. $6z + 3 > 16$
14. $8w - 2 \leq 13$

15. $-8 > \frac{2}{3}t$
16. $-4 \leq \frac{1}{3}p$
17. $\frac{3}{4}(8y - 9) < 3$

18. $\frac{2}{5}(2x - 1) > 10$
19. $0.8a - 0.5 \leq 0.3a - 11$
20. $0.2w - 0.7 < 0.4 - 0.9w$

21. $-5x + 7 < 22$
22. $-3w - 6 > 9$
23. $\frac{-5}{6}x \leq \frac{-3}{4}$

24. $\frac{-3}{2}y > \frac{-21}{16}$
25. $\frac{3p - 1}{-2} > 5$
26. $\frac{3k - 2}{-5} \leq 4$

27. $0.2t + 1 > 2.4t - 10$
28. $20 \leq 8 - \frac{1}{3}x$
29. $3 - 4(y + 2) \leq 6 + 4(2y + 1)$

30. $1 + 4(b - 2) < 2(b - 5) + 4$
31. $7.2k - 5.1 \geq 5.7$
32. $6h - 2.92 \leq 16.58$

33. $-6p - 1 > 17$
34. $-4y + 1 \leq -11$
35. $\frac{3}{4}x - 8 \leq 1$

36. $\frac{-2}{5}a - 3 > 5$
37. $-1.2b - 0.4 \geq -0.4b$
38. $-0.4t + 1.2 < -2$

39. $\frac{-3}{4}e - \frac{-5}{4} \geq 2c$
40. $\frac{2}{3}y - \frac{1}{3} \geq \frac{1}{2}y$
41. $4 - 4(y - 2) < -5y + 6$

42. $6 - 6(k - 3) \geq -4k + 12$
43. $-6(2x + 1) < 5 - (x - 4) - 6x$
44. $2(4p + 3) - p \leq 5 + 3(p - 3)$

45. $6a - (9a + 1) - 3(a - 1) \geq 2$
46. $8(q + 1) - (2q + 1) + 5 > 12$
Chapter 1  Linear Equations and Inequalities in One Variable

Concept 2: Applications of Inequalities

47. Nadia received quiz grades of 80%, 86%, 73%, and 91%. (See Example 5.)  
   a. What grade would she need to make on the fifth quiz to get a B average, that is, at least 80% but less than 90%?  
   b. Is it possible for Nadia to get an A average for her quizzes (at least 90%)?

48. Ty received test grades of 78%, 75%, 71%, 83%, and 73%.  
   a. What grade would he need to make on the sixth test to get a C if a C is at least 75% but less than 80%?  
   b. Is it possible for Ty to get a B or better for his test average (at least 80%)?

For Exercises 49–52, use the graph that shows the average height for boys based on age. Let $a$ represent a boy’s age (in years) and let $h$ represent his height (in inches). (See Example 6.)

49. Determine the age range for which the average height of boys is at least 51 in.
50. Determine the age range for which the average height of boys is greater than or equal to 41 in.
51. Determine the age range for which the average height of boys is no more than 46 in.
52. Determine the age range for which the average height of boys is at most 53.5 in.

53. Nolvia sells copy machines, and her salary is $25,000 plus a 4% commission on sales. The equation $S = 25,000 + 0.04x$ represents her salary $S$ in dollars in terms of her total sales $x$ in dollars.  
   a. How much money in sales does Nolvia need to earn a salary that exceeds $40,000?  
   b. How much money in sales does Nolvia need to earn a salary that exceeds $80,000?  
   c. Why is the money in sales required to earn a salary of $80,000 more than twice the money in sales required to earn a salary of $40,000?

54. The amount of money $A$ in a savings account depends on the principal $P$, the interest rate $r$, and the time in years $t$ that the money is invested. The equation $A = P + Prt$ shows the relationship among the variables for an account earning simple interest. If an investor deposits $5000 at 6\%$ simple interest, the account will grow according to the formula $A = 5000 + 5000(0.065)t$.  
   a. How many years will it take for the investment to exceed $10,000? (Round to the nearest tenth of a year.)  
   b. How many years will it take for the investment to exceed $15,000? (Round to the nearest tenth of a year.)

55. The revenue $R$ for selling $x$ fleece jackets is given by the equation $R = 49.95x$. The cost to produce $x$ jackets is $C = 2300 + 18.50x$. Find the number of jackets that the company needs to sell to produce a profit. (Hint: A profit occurs when revenue exceeds cost.)

56. The revenue $R$ for selling $x$ mountain bikes is $R = 249.95x$. The cost to produce $x$ bikes is $C = 56,000 + 140x$. Find the number of bikes that the company needs to sell to produce a profit.
Expanding Your Skills

For Exercises 57–60, assume \( a > b \). Determine which inequality sign (\( > \) or \( < \)) should be inserted to make a true statement. Assume \( a \neq 0 \) and \( b \neq 0 \).

57. \( a + c \underline{\quad} b + c \), for \( c > 0 \)  
58. \( a + c \underline{\quad} b + c \), for \( c < 0 \)  
59. \( ac \underline{\quad} bc \), for \( c < 0 \)  
60. \( ac \underline{\quad} bc \), for \( c > 0 \)

## Compound Inequalities

### 1. Union and Intersection of Sets

Two or more sets can be combined by the operations of union and intersection.

**DEFINITION** A Union \( B \) and A Intersection \( B \)

The **union** of sets \( A \) and \( B \), denoted \( A \cup B \), is the set of elements that belong to set \( A \) or to set \( B \) or to both sets \( A \) and \( B \).

The **intersection** of two sets \( A \) and \( B \), denoted \( A \cap B \), is the set of elements common to both \( A \) and \( B \).

The concepts of the union and intersection of two sets are illustrated in Figures 1-6 and 1-7.

![Figure 1-6](A \cup B)

- \( A \cup B \)
- \( A \) union \( B \)
- The elements in \( A \) or \( B \) or both

![Figure 1-7](A \cap B)

- \( A \cap B \)
- \( A \) intersection \( B \)
- The elements in \( A \) and \( B \)

### Example 1

**Finding the Union and Intersection of Sets**

Given the sets: \( A = \{a, b, c, d, e, f\} \), \( B = \{a, c, e, g, i, k\} \), \( C = \{g, h, i, j, k\} \)

Find:

a. \( A \cup B \)  
b. \( A \cap B \)  
c. \( A \cap C \)

**Solution:**

a. \( A \cup B = \{a, b, c, d, e, f, g, i, k\} \)

The union of \( A \) and \( B \) includes all the elements of \( A \) along with all the elements of \( B \). Notice that the elements \( a, c, \) and \( e \) are not listed twice.

b. \( A \cap B = \{a, c, e\} \)

The intersection of \( A \) and \( B \) includes only those elements that are common to both sets.
Finding the Union and Intersection of Sets

Given the sets:

\( A = \{x | x < 3\} \)

\( B = \{x | x \geq -2\} \)

\( C = \{x | x \geq 5\} \)

Graph the following sets. Then express each set in interval notation.

a. \( A \cap B \)

b. \( A \cup C \)

c. \( A \cap C = \{\} \) (the empty set) 
Because \( A \) and \( C \) share no common elements, the intersection of \( A \) and \( C \) is the empty set (also called the null set).

**Skill Practice** 
Given: \( A = \{r, s, t, u, v, w\} \)

\( B = \{s, v, w, y, z\} \)

\( C = \{x, y, z\} \)

Find: 1. \( B \cup C \) 
2. \( A \cap B \) 
3. \( A \cap C \) 

**Example 2** Finding the Union and Intersection of Sets

Given the sets: \( A = \{x | x < 3\} \)

\( B = \{x | x \geq -2\} \)

\( C = \{x | x \geq 5\} \)

Graph the following sets. Then express each set in interval notation.

a. \( A \cap B \)

b. \( A \cup C \)

**Solution:**

It is helpful to visualize the graphs of individual sets on the number line before taking the union or intersection.

a. Graph of \( A = \{x | x < 3\} \)

Graph of \( B = \{x | x \geq -2\} \)

Graph of \( A \cap B \) (the “overlap”)

Interval notation: \([-2, 3)\)

Note that the set \( A \cap B \) represents the real numbers greater than or equal to \(-2\) and less than \(3\). This relationship can be written more concisely as a compound inequality: \(-2 \leq x < 3\). We can interpret this inequality as “\(x\) is between \(-2\) and \(3\), including \(x = -2\).”

b. Graph of \( A = \{x | x < 3\} \)

Graph of \( C = \{x | x \geq 5\} \)

Graph of \( A \cup C \) 

Interval notation: \((\infty, 3) \cup [5, \infty)\)

\(A \cup C\) includes all elements from set \(A\) along with the elements from set \(C\).

**Skill Practice** Given the sets: \( A = \{x | x \geq -1\} \)

\( B = \{x | x < 4\} \)

\( C = \{x | x \geq 9\} \), determine the union or intersection and express the answer in interval notation.

4. \( A \cap B \) 
5. \( B \cup C \) 

In Example 3, we find the union and intersection of sets expressed in interval notation.
Example 3  Finding the Union and Intersection of Two Intervals

Find the union or intersection as indicated. Write the answer in interval notation.

a. \((-\infty, -2) \cup [-4, 3)\)  \hspace{1cm} b. \((-\infty, -2) \cap [-4, 3)\)

Solution:

a. \((-\infty, -2) \cup [-4, 3)\) To find the union, graph each interval separately. The union is the collection of real numbers that lie in the first interval, the second interval, or both intervals.

\[
\begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\((-\infty, -2)\)

\[
\begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\([-4, 3)\)

The union consists of all real numbers in the red interval along with the real numbers in the blue interval: \((-\infty, 3)\)

The union is \((-\infty, 3)\).

b. \((-\infty, -2) \cap [-4, 3)\)

\[
\begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\((-\infty, -2)\)

\[
\begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\([-4, 3)\)

The intersection is the “overlap” of the two intervals: \([-4, -2)\).

The intersection is \([-4, -2)\).

Skill Practice  Find the union or intersection. Write the answer in interval notation.

6. \((-\infty, -5) \cup (-7, 0)\)  \hspace{1cm} 7. \((-\infty, -5) \cap (-7, 0)\)

2. Solving Compound Inequalities: And

The solution to two inequalities joined by the word \textit{and} is the intersection of their solution sets. The solution to two inequalities joined by the word \textit{or} is the union of their solution sets.

PROCEDURE  Solving a Compound Inequality

\textbf{Step 1}  Solve and graph each inequality separately.

\textbf{Step 2}  
- If the inequalities are joined by the word \textit{and}, find the intersection of the two solution sets.
- If the inequalities are joined by the word \textit{or}, find the union of the two solution sets.

\textbf{Step 3}  Express the solution set in interval notation or in set-builder notation.

Answers

6. \((-\infty, 0)\)  \hspace{1cm} 7. \((-7, -5)\)
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As you work through the examples in this section, remember that multiplying or dividing an inequality by a negative factor reverses the direction of the inequality sign.

**Example 4  Solving a Compound Inequality: And**

Solve the compound inequality.

\[-2x < 6 \quad \text{and} \quad x + 5 \leq 7\]

**Solution:**

\[-2x < 6 \quad \text{and} \quad x + 5 \leq 7\]

Solve each inequality separately.

\[-\frac{2x}{2} > \frac{6}{2} \quad \text{and} \quad x \leq 2\]

\[x > -3 \quad \text{and} \quad x \leq 2\]

\[\{x|x > -3\}\]

\[\{x|x \leq 2\}\]

Take the intersection of the solution sets: \(\{x|-3 < x \leq 2\}\)

The solution is \(\{x|-3 < x \leq 2\}\), or equivalently in interval notation, \((-3, 2]\).

**Skill Practice** Solve the compound inequality.

8. \(5x + 2 \geq -8\) and \(-4x > -24\)

**Example 5  Solving a Compound Inequality: And**

Solve the compound inequality.

\[4.4a + 3.1 < -12.3 \quad \text{and} \quad -2.8a + 9.1 < -6.3\]

**Solution:**

\[4.4a + 3.1 < -12.3 \quad \text{and} \quad -2.8a + 9.1 < -6.3\]

\[4.4a < -15.4 \quad \text{and} \quad -2.8a < -15.4\]

Solve each inequality separately.

\[-\frac{4.4a}{4.4} < \frac{-15.4}{4.4} \quad \text{and} \quad -\frac{-2.8a}{-2.8} > \frac{-15.4}{-2.8}\]

\[a < -3.5 \quad \text{and} \quad a > 5.5\]

\[\{a|a < -3.5\}\]

\[\{a|a > 5.5\}\]

The intersection of the solution sets is the empty set: \(\{\}\)
There are no real numbers that are simultaneously less than $-3.5$ and greater than $5.5$. There is no solution.

The solution set is { }.

**Skill Practice** Solve the compound inequality.

9. $3.2y - 2.4 > 16.8$ and $-4.1y \geq 8.2$

---

**Example 6** Solving a Compound Inequality: And

Solve the compound inequality.

\[
\frac{2}{3}x \leq 6 \quad \text{and} \quad \frac{1}{2}x < 1
\]

**Solution:**

\[
\frac{2}{3}x \leq 6 \quad \text{and} \quad \frac{1}{2}x < 1
\]

\[\frac{3}{2} \left( \frac{2}{3}x \right) \geq \frac{3}{2}(6) \quad \text{and} \quad -2 \left( \frac{1}{2}x \right) > -2(1)\]

\[x \geq -9 \quad \text{and} \quad x > -2\]

The solution set is \( \{x | x > -2\} \), or in interval notation, \((-2, \infty)\).

**Skill Practice** Solve the compound inequality.

10. \( \frac{1}{4}z < \frac{5}{8} \) and \( \frac{1}{2}z + 1 \geq 3 \)

---

3. **Solving Inequalities of the Form** \( a < x < b \)

An inequality of the form \( a < x < b \) is a type of **compound inequality**, one that defines two simultaneous conditions on \( x \).

The solution set to the compound inequality \( a < x < b \) is the **intersection** of the solution sets to the inequalities \( a < x \) and \( x < b \).
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Example 7  Solving an Inequality of the Form $a < x < b$

Solve the inequality.  $-4 < 3x + 5 \leq 10$

**Solution:**

Set up the intersection of two inequalities.

\[-4 < 3x + 5 \quad \text{and} \quad 3x + 5 \leq 10\]

\[-9 < 3x \quad \text{and} \quad 3x \leq 5\]

\[-9 < 3x \quad \text{and} \quad 3x \leq 5\]

\[-3 < x \quad \text{and} \quad x \leq \frac{5}{3}\]

\[-3 < x \leq \frac{5}{3}\]

The solution is $\{x \mid -3 < x \leq \frac{5}{3}\}$, or equivalently in interval notation, $(-3, \frac{5}{3}]$.

**Skill Practice**  Solve the inequality.

11. $-6 \leq 2x - 5 < 1$

To solve an inequality of the form $a < x < b$, we can also work with the inequality as a “three-part” inequality and isolate $x$. This is demonstrated in Example 8.

Example 8  Solving an Inequality of the Form $a < x < b$

Solve the inequality.  $2 \geq \frac{p - 2}{-3} \geq -1$

**Solution:**

Isolate the variable in the middle part.

\[2 \geq \frac{p - 2}{-3} \geq -1\]

\[-3(2) \leq -3 \left( \frac{p - 2}{-3} \right) \leq -3(-1)\]

\[-6 \leq p - 2 \leq 3\]

\[-6 + 2 \leq p - 2 + 2 \leq 3 + 2\]

\[-4 \leq p \leq 5\]

The solution is $\{p \mid -4 \leq p \leq 5\}$, or equivalently in interval notation $[-4, 5]$.

**Skill Practice**  Solve the inequality.

12. $8 > \frac{t + 4}{-2} > -5$
**4. Solving Compound Inequalities: Or**

In Examples 9 and 10, we solve compound inequalities that involve inequalities joined by the word “or.” In such a case, the solution to the compound inequality is the union of the solution sets of the individual inequalities.

**Example 9** Solving a Compound Inequality: Or

Solve the compound inequality.

\[-3y - 5 > 4 \quad \text{or} \quad 4 - y \leq 6\]

**Solution:**

\[-3y - 5 > 4 \quad \text{or} \quad 4 - y \leq 6\]

\[-3y > 9 \quad \text{or} \quad -y \leq 2\]

\[-3y < 9 \quad \text{or} \quad -y \geq 2\]

\[-3y < 9 \quad \text{or} \quad -y \geq 2\]

\[y < -3 \quad \text{or} \quad y \geq -2\]

\(\{y \mid y < -3\} \quad \{y \mid y \geq -2\}\)

The solution is \(\{y \mid y < -3 \text{ or } y \geq -2\}\) or, equivalently in interval notation, \((-\infty, -3) \cup [2, \infty)\).

**Skill Practice** Solve the compound inequality.

13. \(-10t - 8 \leq 12 \text{ or } 3t - 6 > 3\)

**Example 10** Solving a Compound Inequality: Or

Solve the compound inequality.

\[4x + 3 < 16 \quad \text{or} \quad -2x < 3\]

**Solution:**

\[4x + 3 < 16 \quad \text{or} \quad -2x < 3\]

\[4x < 13 \quad \text{or} \quad x > -\frac{3}{2}\]

\[x < \frac{13}{4} \quad \text{or} \quad x > -\frac{3}{2}\]

\[\{t \mid t \leq -2 \text{ or } t > 3\}; \quad (-\infty, -2) \cup (3, \infty)\]

**Answer**

13. \(\{t \mid t \leq -2 \text{ or } t > 3\}; \quad (-\infty, -2) \cup (3, \infty)\)
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The union of the solution sets is \( \{x \mid x \text{ is a real number}\} \), or equivalently, \( (-\infty, \infty) \).

Skill Practice  Solve the compound inequality.

14. \( x - 7 > -2 \) or \( -6x > -48 \)

5. Applications of Compound Inequalities

Compound inequalities are used in many applications, as shown in Examples 11 and 12.

Example 11  Translating Compound Inequalities

The normal level of thyroid-stimulating hormone (TSH) for adults ranges from 0.4 to 4.8 microunits per milliliter (\( \mu U/mL \)), inclusive. Let \( x \) represent the amount of TSH measured in microunits per milliliter.

a. Write an inequality representing the normal range of TSH, inclusive.

b. Write a compound inequality representing abnormal TSH levels.

Solution:

a. \( 0.4 \leq x \leq 4.8 \)

b. \( x < 0.4 \) or \( x > 4.8 \)

Skill Practice  The length of a normal human pregnancy, \( w \), is from 37 to 41 weeks, inclusive.

15. Write an inequality representing the normal length of a pregnancy.

16. Write a compound inequality representing an abnormal length for a pregnancy.

TIP:

- In mathematics, the word “between” means strictly between two values. That is, the endpoints are excluded. 
  Example: \( x \) is between 4 and 10 \( \Rightarrow (4, 10) \).
- If the word “inclusive” is added to the statement, then we include the endpoints. 
  Example: \( x \) is between 4 and 10, inclusive \( \Rightarrow [4, 10] \).

Answers

14. \( \{x \mid x \text{ is a real number}\}; (-\infty, \infty) \)

15. \( 37 \leq w \leq 41 \)

16. \( w < 37 \) or \( w > 41 \)
Example 12  Translating and Solving a Compound Inequality

The sum of a number and 4 is between −5 and 12. Find all such numbers.

Solution:
Let $x$ represent a number.

- $-5 < x + 4 < 12$
- $-5 - 4 < x + 4 - 4 < 12 - 4$
- $-9 < x < 8$

The number may be any real number between −9 and 8: \{x | -9 < x < 8\}.

Skill Practice
17. The sum of twice a number and 11 is between 21 and 31. Find all such numbers.

Example 12

Answer
17. Any real number between 5 and 10: \{n | 5 < n < 10\}

Section 1.5  Practice Exercises

Study Skills Exercises
1. Which activities might you try when working in a study group to help you learn and understand the material?
   - [ ] Quiz one another by asking one another questions.
   - [ ] Practice teaching one another.
   - [ ] Share and compare class notes.
   - [ ] Support and encourage one another.
   - [ ] Work together on exercises and sample problems.

2. Define the key terms.
   - a. Compound inequality
   - b. Intersection
   - c. Union

Review Exercises
For Exercises 3–6, solve the linear inequality. Write the solution in interval notation.

3. $-6u + 8 > 2$
4. $2 - 3z \geq -4$
5. $-12 \leq \frac{3}{4}p$
6. $5 > \frac{1}{3}w$

Concept 1: Union and Intersection of Sets
7. Given: $M = \{-3, -1, 1, 3, 5\}$ and $N = \{-4, -3, -2, -1, 0\}$ (See Example 1.)
   List the elements of the following sets:
   - a. $M \cap N$
   - b. $M \cup N$

8. Given: $P = \{a, b, c, d, e, f, g, h, i\}$ and $Q = \{a, e, i, o, u\}.$
   List the elements of the following sets.
   - a. $P \cap Q$
   - b. $P \cup Q$
Chapter 1  Linear Equations and Inequalities in One Variable

For Exercises 9–20, refer to the sets $A, B, C,$ and $D.$ Determine the union or intersection as indicated. Express the answer in interval notation, if possible. (See Example 2)

$A = \{x|x < -4\}$, $B = \{x|x > 2\}$, $C = \{x|x \geq -7\}$, $D = \{x|0 \leq x < 5\}$

9. $A \cap C$  
10. $B \cap C$  
11. $A \cup B$
12. $A \cup D$  
13. $A \cap B$  
14. $A \cap D$
15. $B \cup C$  
16. $B \cup D$  
17. $C \cap D$
18. $B \cap D$  
19. $C \cup D$  
20. $A \cup C$

For Exercises 21–26, find the intersection and union of sets as indicated. Write the answers in interval notation. (See Example 3.)

21. a. $(-2, 5) \cap [-1, \infty)$  
   b. $(-2, 5) \cup [-1, \infty)$
22. a. $(-\infty, 4) \cap [-1, 5)$
   b. $(-\infty, 4) \cup [-1, 5)$
23. a. $\left(\frac{5}{2}, 3\right) \cap \left(-1, \frac{9}{2}\right)$
   b. $\left(\frac{5}{2}, 3\right) \cup \left(-1, \frac{9}{2}\right)$
24. a. $(3.4, 1.6) \cap (-2.2, 4.1)$
   b. $(3.4, 1.6) \cup (-2.2, 4.1)$
25. a. $(-4, 5] \cap (0, 2]$  
   b. $(-4, 5] \cup (0, 2]$  
26. a. $[-1, 5] \cap (0, 3)$
   b. $[-1, 5] \cup (0, 3)$

Concept 2: Solving Compound Inequalities: And

For Exercises 27–36, solve the inequality and graph the solution. Write the answer in interval notation. (See Examples 4–6.)

27. $y - 7 \geq -9$ and $y + 2 \leq 5$
28. $a + 6 > -2$ and $5a < 30$
29. $2t + 7 < 19$ and $5t + 13 > 28$
30. $5p + 2p \geq -21$ and $-9p + 3p \geq -24$
31. $2.1k - 1.1 \leq 0.6k + 1.9$ and $0.3k - 1.1 < -0.1k + 0.9$
32. $0.6w + 0.1 > 0.3w - 1.1$ and $2.3w + 1.5 \geq 0.3w + 6.5$
33. $\frac{2}{3}(2p - 1) \geq 10$ and $\frac{4}{5}(3p + 4) \geq 20$
34. $\frac{5}{2}(a + 2) < -6$ and $\frac{3}{4}(a - 2) < 1$
35. $-2 < -x - 12$ and $-14 < 5(x - 3) + 6x$
36. $-8 \geq -3y - 2$ and $3(y - 7) + 16 > 4y$

Concept 3: Solving Inequalities of the Form $a < x < b$

37. Write $-4 \leq t < \frac{1}{2}$ as two separate inequalities.
38. Write $-2.8 < y \leq 15$ as two separate inequalities.
39. Explain why $6 < x < 2$ has no solution.
40. Explain why $4 < t < 1$ has no solution.
41. Explain why $-5 > y > -2$ has no solution.
42. Explain why $-3 > w > -1$ has no solution.
Section 1.5 Compound Inequalities

For Exercises 43–54, solve the inequality and graph the solution set. Write the answer in interval notation. (See Examples 7–8.)

43. $0 \leq 2h - 5 < 9$
44. $-6 < 3k - 9 \leq 0$
45. $-1 < \frac{a}{6} \leq 1$
46. $-3 \leq \frac{1}{2}x < 0$
47. $-\frac{2}{3} < \frac{y - 4}{-6} < \frac{1}{3}$
48. $\frac{1}{3} \geq \frac{t - 4}{-3} > -2$
49. $5 \leq -3x - 2 \leq 8$
50. $-1 < -2x + 4 \leq 5$
51. $12 > 6x + 3 \geq 0$
52. $-4 \geq 2x - 5 > -7$
53. $-0.2 < 2.6 + 7t < 4$
54. $-1.5 < 0.1x \leq 8.1$

Concept 4: Solving Compound Inequalities: Or

For Exercises 55–64, solve the inequality and graph the solution set. Write the answer in interval notation. (See Examples 9–10.)

55. $2y - 1 \geq 3$ or $y < -2$
56. $x < 0$ or $3x + 1 \geq 7$
57. $1 > 6z - 8$ or $8z - 6 \leq 10$
58. $22 > 4t - 10$ or $7 > 2t - 5$
59. $5(x - 1) \geq -5$ or $5 - x \leq 11$
60. $-p + 7 \geq 10$ or $3(p - 1) \leq 12$
61. $\frac{5}{3}v \leq 5$ or $-v - 6 < 1$
62. $\frac{3}{8}u + 1 > 0$ or $-2u \geq -4$
63. $0.5w + 5 < 2.5w - 4$ or $0.3w \leq -0.1w - 1.6$
64. $1.25a + 3 \leq 0.5a - 6$ or $2.5a - 1 \geq 9 - 1.5a$

Mixed Exercises

For Exercises 65–74, solve the inequality. Write the answer in interval notation.

65. a. $3x - 5 < 19$ and $-2x + 3 < 23$
   b. $3x - 5 < 19$ or $-2x + 3 < 23$
66. a. $0.5(6x + 8) > 0.8x - 7$ and $4(x + 1) < 7.2$
   b. $0.5(6x + 8) > 0.8x - 7$ or $4(x + 1) < 7.2$
67. a. $8x - 4 \geq 6.4$ or $0.3(x + 6) \leq -0.6$
   b. $8x - 4 \geq 6.4$ and $0.3(x + 6) \leq -0.6$
68. a. $-2r + 4 \leq -8$ or $3r + 5 \leq 8$
   b. $-2r + 4 \leq -8$ and $3r + 5 \leq 8$
69. $-4 \leq \frac{2 - 4x}{3} < 8$
70. $-1 < \frac{3 - x}{2} \leq 0$
Chapter 1  Linear Equations and Inequalities in One Variable

71. \(5 \geq -4(t - 3) + 3t \text{ or } 6 < 12t + 8(4 - t)\)
72. \(3 > -(w - 3) + 4w \text{ or } -5 \geq -3(w - 5) + 6w\)
73. \(- \frac{x + 3}{2} > \frac{4 + x}{5} \text{ or } \frac{1 - x}{4} > \frac{2 - x}{3}\)
74. \(\frac{y - 7}{-3} < \frac{1}{4} \text{ or } \frac{y + 1}{-2} > \frac{1}{3}\)

Concept 5: Applications of Compound Inequalities

75. The normal number of white blood cells for human blood is between 4800 and 10,800 cells per cubic millimeter, inclusive. Let \(x\) represent the number of white blood cells per cubic millimeter. (See Example 11.)
   a. Write an inequality representing the normal range of white blood cells per cubic millimeter.
   b. Write a compound inequality representing abnormal levels of white blood cells per cubic millimeter.

76. Normal hemoglobin levels in human blood for adult males are between 13 and 16 grams per deciliter (g/dL), inclusive. Let \(x\) represent the level of hemoglobin measured in grams per deciliter.
   a. Write an inequality representing normal hemoglobin levels for adult males.
   b. Write a compound inequality representing abnormal levels of hemoglobin for adult males.

77. The normal number of platelets in human blood is between 200,000 and 350,000 platelets per cubic millimeter, inclusive. Let \(x\) represent the number of platelets per cubic millimeter.
   a. Write an inequality representing a normal platelet count per cubic millimeter.
   b. Write a compound inequality representing abnormal platelet counts per cubic millimeter.

78. Normal hemoglobin levels in human blood for adult females are between 12 and 15 g/dL, inclusive. Let \(x\) represent the level of hemoglobin measured in grams per deciliter.
   a. Write an inequality representing normal hemoglobin levels for adult females.
   b. Write a compound inequality representing abnormal levels of hemoglobin for adult females.

79. Twice a number is between \(-3\) and 12. Find all such numbers. (See Example 12.)

80. The difference of a number and 6 is between 0 and 8. Find all such numbers.

81. One plus twice a number is either greater than 5 or less than \(-1\). Find all such numbers.

82. One-third of a number is either less than \(-2\) or greater than 5. Find all such numbers.

83. Amy knows from reading her syllabus in intermediate algebra that the average of her chapter tests accounts for 80\% (0.8) of her overall course grade. She also knows that the final exam counts as 20\% (0.2) of her grade.
   a. Suppose that the average of Amy’s chapter tests is 92\%. Determine the range of grades that she would need on her final exam to get an “A” in the class. (Assume that a grade of “A” is obtained if Amy’s overall average is 90\% or better.)
   b. Determine the range of grades that Amy would need on her final exam to get a “B” in the class. (Assume that a grade of “B” is obtained if Amy’s overall average is at least 80\% but less than 90\%).
84. Robert knows from reading his syllabus in intermediate algebra that the average of his chapter tests accounts for 60% (0.6) of his overall course grade. He also knows that the final exam counts as 40% (0.4) of his grade.

a. Suppose that the average of Robert’s chapter tests is 89%. Determine the range of grades that he would need on his final exam to get an “A” in the class. (Assume that a grade of “A” is obtained if Robert’s overall average is 90% or better.)

b. Determine the range of grades that Robert would need on his final exam to get a “B” in the class. (Assume that a grade of “B” is obtained if Robert’s overall average is at least 80% but less than 90%).

85. The average high and low temperatures for Vancouver, British Columbia, in January are 5.6°C and 0°C, respectively. The formula relating Celsius temperatures to Fahrenheit temperatures is given by $C = \frac{5}{9}(F - 32)$. Convert the inequality $0.0 \leq C \leq 5.6$ to an equivalent inequality using Fahrenheit temperatures.

86. For a day in July, the temperatures in Austin, Texas, ranged from 20°C to 29°C. The formula relating Celsius temperatures to Fahrenheit temperatures is given by $C = \frac{5}{9}(F - 32)$. Convert the inequality $20 \leq C \leq 29$ to an equivalent inequality using Fahrenheit temperatures.

### Absolute Value Equations

#### 1. Solving Absolute Value Equations

An equation of the form $|x| = a$ is called an absolute value equation. For example, consider the equation $|x| = 4$. From the definition of absolute value, the solutions are found by solving the equations $x = 4$ and $-x = 4$. This gives the equivalent equations $x = 4$ and $x = -4$.

Also recall from Section R.3 that the absolute value of a number is its distance from zero on the number line. Therefore, geometrically, the solutions to the equation $|x| = 4$ are the values of $x$ that are 4 units from zero on the number line (Figure 1-8).

![Figure 1-8](image)

**PROCEDURE Solving Absolute Value Equations of the Form $|x| = a$**

If $a$ is a real number, then

- If $a \geq 0$, the solutions to the equation $|x| = a$ are given by $x = a$ and $x = -a$.
- If $a < 0$, there is no solution to the equation $|x| = a$. 
Chapter 1  Linear Equations and Inequalities in One Variable

To solve an absolute value equation of the form $|x| = a$ ($a \geq 0$), rewrite the equation as $x = a$ or $x = -a$.

**Example 1**  Solving Absolute Value Equations

Solve the absolute value equations.

a. $|x| = 5$  
   b. $|w| - 2 = 12$  
   c. $|p| = 0$  
   d. $|x| = -6$

**Solution:**

a. $|x| = 5$  
   The equation is in the form $|x| = a$, where $a = 5$.
   $x = 5$  or  $x = -5$  
   The solution set is $\{5, -5\}$.

b. $|w| - 2 = 12$  
   Isolate the absolute value to write the equation in the form $|w| = a$.
   $|w| = 14$  
   $w = 14$  or  $w = -14$  
   The solution set is $\{14, -14\}$.

c. $|p| = 0$  
   Rewrite as two equations. Notice that the second equation $p = -0$ is the same as the first equation. Intuitively, $p = 0$ is the only number whose absolute value equals 0.
   $p = 0$  or  $p = -0$  
   The solution set is $\{0\}$.

d. $|x| = -6$  
   No solution, $\{\}$

**Skill Practice**  Solve the absolute value equations.

1. $|y| = 7$  
2. $|v| + 6 = 10$  
3. $|w| = 0$  
4. $|z| = -12$

We have solved absolute value equations of the form $|x| = a$. Notice that $x$ can represent any algebraic quantity. For example, to solve the equation $|2w - 3| = 5$, we still rewrite the absolute value equation as two equations. In this case, we set the quantity $2w - 3$ equal to 5 and to $-5$, respectively.

$|2w - 3| = 5$

$2w - 3 = 5$  or  $2w - 3 = -5$

**Answers**

1. $\{7, -7\}$  
2. $\{4, -4\}$  
3. $\{0\}$  
4. $\{}$
PROCEDURE  Solving an Absolute Value Equation

Step 1 Isolate the absolute value. That is, write the equation in the form \(|x| = a\), where \(a\) is a real number.

Step 2 If \(a < 0\), there is no solution.

Step 3 Otherwise, if \(a \geq 0\), rewrite the absolute value equation as 
\[ x = a \text{ or } x = -a. \]

Step 4 Solve the individual equations from step 3.

Step 5 Check the answers in the original absolute value equation.

Example 2  Solving an Absolute Value Equation

Solve the equation. \(|2w - 3| = 5\)

Solution:  
\[ |2w - 3| = 5 \]

The equation is already in the form \(|x| = a\), where \(x = 2w - 3\).

\[ 2w - 3 = 5 \quad \text{or} \quad 2w - 3 = -5 \]

Rewrite as two equations.

\[ 2w = 8 \quad \text{or} \quad 2w = -2 \]

Solve each equation.

\[ w = 4 \quad \text{or} \quad w = -1 \]

Check: \(w = 4\)  
\[ |2w - 3| = 5 \]
\[ |2(4) - 3| = 5 \]
\[ |8 - 3| = 5 \]
\[ |5| = 5 \checkmark \]

Check: \(w = -1\)  
\[ |2w - 3| = 5 \]
\[ |2(-1) - 3| = 5 \]
\[ |-2 - 3| = 5 \]
\[ |-5| = 5 \checkmark \]

The solution set is \(\{4, -1\}\).

Skill Practice  Solve the equation.
5. \(|4x + 1| = 9\)

Example 3  Solving an Absolute Value Equation

Solve the equation. \(|2c - 5| + 6 = 2\)

Solution:  
\[ |2c - 5| + 6 = 2 \]
\[ |2c - 5| = -4 \]

Isolate the absolute value. The equation is in the form \(|x| = a\), where \(x = 2c - 5\) and \(a = -4\).

Because \(a < 0\), there is no solution.

No solution, \(\{\}\)

There are no numbers \(c\) that will make an absolute value equal to a negative number.

Skill Practice  Solve the equation.
6. \(|3z + 10| + 3 = 1\)

Avoiding Mistakes
Always isolate the absolute value first. Otherwise you will get answers that do not check.

Answers
5. \(\{2, -\frac{5}{2}\}\) 6. No solution, \(\{\}\)
Example 4  Solving an Absolute Value Equation

Solve the equation.

\[-2 \left| \frac{2}{5}p + 3 \right| - 7 = -19\]

Solution:

\[-2 \left| \frac{2}{5}p + 3 \right| = -12\]

Isolate the absolute value.

\[-2 \left( \frac{2}{5}p + 3 \right) = -12\]

Divide both sides by -2.

\[\frac{2}{5}p + 3 = 6\]

Rewrite as two equations.

or \[\frac{2}{5}p + 3 = -6\]

\[2p + 15 = 30\]

Multiply by 5 to clear fractions.

or \[2p + 15 = -30\]

or \[2p = 15\]

Both solutions check in the original equation.

or \[2p = -45\]

The solution set is \[\left\{ \frac{15}{2}, -\frac{45}{2} \right\}\].

Skill Practice  Solve the equation.

7. \[\left| \frac{3}{2}a + 1 \right| + 2 = 14\]

Example 5  Solving an Absolute Value Equation

Solve the equation.

\[6.9 = |4.1 - p| + 6.9\]

Solution:

\[6.9 = |4.1 - p| + 6.9\]

First write the absolute value on the left. Then subtract 6.9 from both sides to write the equation in the form \(|x| = a\).

\[|4.1 - p| = 0\]

Isolate the absolute value.

\[4.1 - p = 0\]  or  \[4.1 - p = -0\]

Rewrite as two equations. Notice that the equations are the same.

Answer

7. \[\left\{ \frac{2}{3}, -\frac{10}{3} \right\}\]
Section 1.6 Absolute Value Equations

Subtract 4.1 from both sides.

Check: \( p = 4.1 \)
- \(|4.1 - p| + 6.9 = 6.9\)
- \(|4.1 - 4.1| + 6.9 \pm 6.9\)
- \(|0| + 6.9 \pm 6.9\)

The solution set is \( \{4.1\} \).

Skill Practice Solve the equation.
8. \(-3.5 = |1.2 + x| - 3.5\)

2. Solving Equations Containing Two Absolute Values

Some equations have two absolute values such as \(|x| = |y|\). If two quantities have the same absolute value, then the quantities are equal or the quantities are opposites.

**PROPERTY Equality of Absolute Values**

\(|x| = |y|\) implies that \( x = y \) or \( x = -y \).

**Example 6** Solving an Equation Having Two Absolute Values

Solve the equation. \(|2w - 3| = |5w + 1|\)

Solution:

\[ |2w - 3| = |5w + 1| \]

- \(2w - 3 = 5w + 1\) or \(2w - 3 = -(5w + 1)\)
- \(2w - 3 = 5w + 1\) or \(2w - 3 = -5w - 1\)
- \(-3w - 3 = 1\) or \(7w - 3 = -1\)
- \(-3w = 4\) or \(7w = 2\)
- \(w = -\frac{4}{3}\) or \(w = \frac{2}{7}\)

Both values check in the original equation.

The solution set is \( \left\{ -\frac{4}{3}, \frac{2}{7} \right\} \).

Skill Practice Solve the equation.
9. \(|3 - 2x| = |3x - 1|\)

Answers
8. \(\{-1.2\}\)
9. \(\left\{ -\frac{4}{3}, \frac{2}{7} \right\}\)
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Example 7 Solving an Equation Having Two Absolute Values

Solve the equation. \( |x - 4| = |x + 8| \)

Solution:
\[
|x - 4| = |x + 8| \quad \text{Rewrite as two equations,}
\]
\[
x - 4 = x + 8 \quad \text{or} \quad x - 4 = -(x + 8)
\]
\[
x - 4 = 8 \quad \text{or} \quad x - 4 = -x - 8
\]
\[
\text{contradiction}
\]
\[
2x - 4 = -8
\]
\[
x = -2 \quad x = -2 \checkmark \text{checks in the original equation.}
\]

The solution set is \{-2\}.

Skill Practice Solve the equation.
10. \(|4t + 3| = |4t - 5|\)

Section 1.6 Practice Exercises

Study Skills Exercises

1. One way to know that you really understand a concept is to explain it to someone else. In your own words, explain how to solve an absolute value equation.

2. Define the key term absolute value equation.

Review Exercises

For Exercises 3–6, solve the inequalities. Write the answers in interval notation.

3. \(3(a + 2) - 6 > 2 \quad \text{and} \quad -2(a - 3) + 14 > -3\)

4. \(3x - 5 \geq 7x + 3 \quad \text{or} \quad 2x - 1 \leq 4x - 5\)

5. \(5 \geq \frac{x - 4}{-2} > -3\)

6. \(4 \leq \frac{1}{3}x - 2 < 7\)

Concept 1: Solving Absolute Value Equations

For Exercises 7–38, solve the absolute value equations. (See Examples 1–5.)

7. \(|p| = 7\)

8. \(|q| = 10\)

9. \(|x| + 5 = 11\)

10. \(|x| - 3 = 20\)

11. \(|y| = \sqrt{2}\)

12. \(|y| = \sqrt{3}\)

13. \(|w| - 3 = -5\)

14. \(|w| + 4 = -8\)

15. \(|3q| = 0\)

16. \(|4p| = 0\)

17. \(|3x - 4| = 8\)

18. \(|4x + 1| = 6\)
Section 1.7 Absolute Value Inequalities

19. 5 = |2x - 4|  
20. 10 = |3x + 7|  
21. \[ \frac{7x}{3} - \frac{1}{3} + 3 = 6 \]  
22. \[ \frac{w}{2} + \frac{3}{2} - 2 = 7 \]  

23. |0.2x - 3.5| = -5.6  
24. |1.81 + 2x| = -2.2  
25. 1 = -4 + \[ \frac{2}{1} - \frac{1}{4}w \]  
26. -12 = -6 - |6 - 2x|  

27. 10 = 4 + |2y + 1|  
28. -1 = -|5x + 7|  
29. -2|3b - 7| - 9 = -9  
30. -3|5x + 1| + 4 = 4  

31. -2|x + 3| = 5  
32. -3|x - 5| = 7  
33. 0 = |6x - 9|  
34. 7 = |4k - 6| + 7  

35. \[ -\frac{1}{5} - \frac{1}{2}x = \frac{9}{5} \]  
36. \[ -\frac{1}{6} - \frac{2}{9}h = \frac{1}{2} \]  
37. -3|2 - 6x| + 5 = -10  
38. 5|1 - 2x| - 7 = 3  

Concept 2: Solving Equations Containing Two Absolute Values

For Exercises 39–52, solve the absolute value equations. (See Examples 6–7.)

39. |4x - 2| = |-8|  
40. |3x + 5| = |-5|  
41. |4w + 3| = |2w - 5|  

42. |3y + 1| = |2y - 7|  
43. |2y + 5| = |7 - 2y|  
44. |9a + 5| = |9a - 1|  

45. \[ \frac{4w - 1}{6} = \frac{2w}{3} + \frac{1}{4} \]  
46. \[ \frac{3p + 2}{4} = \frac{1}{2}p - 2 \]  
47. |x + 2| = |-x - 2|  

48. |2y - 3| = |-2y + 3|  
49. |3.5m - 1.2| = |8.5m + 6|  
50. |11.2n + 9| = |7.2n - 2.1|  

Expanding Your Skills

53. Write an absolute value equation whose solution is the set of real numbers 6 units from zero on the number line.  
54. Write an absolute value equation whose solution is the set of real numbers \( \frac{1}{2} \) units from zero on the number line.  

55. Write an absolute value equation whose solution is the set of real numbers \( \frac{2}{3} \) units from zero on the number line.  
56. Write an absolute value equation whose solution is the set of real numbers 9 units from zero on the number line.  

Absolute Value Inequalities

1. Solving Absolute Value Inequalities by Definition

In Section 1.6, we studied absolute value equations in the form \( |x| = a \). In this section, we will solve absolute value inequalities. An inequality in any of the forms \( |x| < a, |x| \leq a, |x| > a, \) or \( |x| \geq a \) is called an absolute value inequality.

Recall that an absolute value represents distance from zero on the real number line. Consider the following absolute value equation and inequalities.

1. \( |x| = 3 \)

   Solution:
   \( x = 3 \) or \( x = -3 \)

   The set of all points 3 units from zero on the number line

   ![Graph of absolute value inequality](image)
Chapter 1  Linear Equations and Inequalities in One Variable

2. \( |x| > 3 \)  
   \( x < -3 \) or \( x > 3 \)  
   Solution:  
   The set of all points more than 3 units from zero  
   \[ 3 \text{ units} \quad 3 \text{ units} \]

3. \( |x| < 3 \)  
   \( -3 < x < 3 \)  
   Solution:  
   The set of all points less than 3 units from zero  
   \[ 3 \text{ units} \quad 3 \text{ units} \]

**PROCEDURE  Solving Absolute Value Equations and Inequalities**

Let \( a \) be a real number such that \( a > 0 \). Then

<table>
<thead>
<tr>
<th>Equation/ Inequality</th>
<th>Solution (Equivalent Form)</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>x</td>
<td>= a )</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
<td>&gt; a )</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
<td>&lt; a )</td>
</tr>
</tbody>
</table>

To solve an absolute value inequality, first isolate the absolute value and then rewrite the absolute value inequality in its equivalent form.

**Example 1  Solving an Absolute Value Inequality**

Solve the inequality.  
\( |3w + 1| - 4 < 7 \)

**Solution:**

\[
|3w + 1| - 4 < 7  
\]

Isolate the absolute value first.  
The inequality is in the form \( |x| < a \), where \( x = 3w + 1 \).

\[
-11 < 3w + 1 < 11  
\]

Rewrite in the equivalent form \(-a < x < a\).

\[
-12 < 3w < 10  
\]

Solve for \( w \).

\[
-4 < w < \frac{10}{3}  
\]

The solution is \( \{-4 < w < \frac{10}{3}\} \), or equivalently in interval notation, \( (-4, \frac{10}{3}) \).

**Skill Practice** Solve the inequality. Write the solution in interval notation.

1. \( |2t + 5| + 2 \leq 11 \)

TIP: Recall that a strict inequality (using the symbols > and <) will have parentheses at the endpoints of the interval form of the solution.
Example 2 Solving an Absolute Value Inequality

Solve the inequality. \[ 3 \leq 1 + \left| \frac{1}{2}t - 5 \right| \]

Solution:

\[ 3 \leq 1 + \left| \frac{1}{2}t - 5 \right| \]

Write the inequality with the absolute value on the left.

\[ 1 + \left| \frac{1}{2}t - 5 \right| \geq 3 \quad \text{Write the inequality with the absolute value on the left.} \]

\[ \left| \frac{1}{2}t - 5 \right| \geq 2 \quad \text{Isolate the absolute value.} \]

The inequality is in the form \(|x| \geq a\), where \(x = \frac{1}{2}t - 5\).

\[ \frac{1}{2}t - 5 \leq -2 \quad \text{or} \quad \frac{1}{2}t - 5 \geq 2 \quad \text{Rewrite in the equivalent form } x \leq -a \text{ or } x \geq a. \]

\[ \frac{1}{2}t \leq 3 \quad \text{or} \quad \frac{1}{2}t \geq 7 \quad \text{Solve the compound inequality.} \]

\[ 2 \left( \frac{1}{2}t \right) \leq 2(3) \quad \text{or} \quad 2 \left( \frac{1}{2}t \right) \geq 2(7) \quad \text{Clear fractions.} \]

\[ t \leq 6 \quad \text{or} \quad t \geq 14 \]

The solution is \( \{ t | t \leq 6 \text{ or } t \geq 14 \} \) or, equivalently in interval notation, \((-\infty, 6] \cup [14, \infty)\).

Skill Practice Solve the inequality. Write the solution in interval notation.

2. \[ 5 < 1 + \left| \frac{1}{3}c - 1 \right| \]

By definition, the absolute value of a real number will always be nonnegative. Therefore, the absolute value of any expression will always be greater than a negative number. Similarly, an absolute value can never be less than a negative number. Let \(a\) represent a positive real number. Then

- The solution to the inequality \(|x| > -a\) is all real numbers, \((-\infty, \infty)\).
- There is no solution to the inequality \(|x| < -a\).

Example 3 Solving Absolute Value Inequalities

Solve the inequalities.

a. \[ |3d - 5| + 7 < 4 \]

b. \[ |3d - 5| + 7 > 4 \]

Solution:

a. \[ |3d - 5| + 7 < 4 \]

Isolate the absolute value. An absolute value expression cannot be less than a negative number. Therefore, there is no solution.

No solution, \( \{ \} \)

Answer

2. \((-\infty, -9) \cup (15, \infty)\)
Chapter 1  Linear Equations and Inequalities in One Variable

b. $|3d - 5| + 7 > 4$

Isolate the absolute value. The inequality is in the form $|x| > a$, where $a$ is negative. An absolute value of any real number is greater than a negative number. Therefore, the solution is all real numbers.

All real numbers, $(-\infty, \infty)$

**Skill Practice** Solve the inequalities.

3. $|4p + 2| + 6 < 2$
4. $|4p + 2| + 6 > 2$

---

**Example 4** Solving Absolute Value Inequalities

Solve the inequalities.

a. $|4x + 2| \geq 0$

The absolute value is already isolated. The absolute value of any real number is nonnegative. Therefore, the solution is all real numbers, $(-\infty, \infty)$.

b. $|4x + 2| > 0$

An absolute value will be greater than zero at all points except where it is equal to zero. That is, the point(s) for which $|4x + 2| = 0$ must be excluded from the solution set.

$|4x + 2| = 0$

$4x + 2 = 0$ or $4x + 2 = -0$  

The second equation is the same as the first.

$4x = -2$

$x = -\frac{1}{2}$

Therefore, exclude $x = -\frac{1}{2}$ from the solution.

The solution is $\{x \mid x \neq -\frac{1}{2}\}$ or equivalently in interval notation, $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$.

c. $|4x + 2| \leq 0$

An absolute value of a number cannot be less than zero. However, it can be equal to zero. Therefore, the only solutions to this inequality are the solutions to the related equation:

$|4x + 2| = 0$  

From part (b), we see that the solution set is $\left\{-\frac{1}{2}\right\}$.

**Skill Practice** Solve the inequalities.

5. $|3x - 1| \geq 0$
6. $|3x - 1| > 0$

---

**Answers**

3. No solution, $\{\}$
4. All real numbers; $(-\infty, \infty)$
5. $(-\infty, \infty)$
6. $\left\{x \mid x \neq \frac{1}{3}\right\}$ or $(-\infty, \frac{1}{3}) \cup \left(\frac{1}{3}, \infty\right)$
Section 1.7 Absolute Value Inequalities

2. Solving Absolute Value Inequalities by the Test Point Method

For the problems in Examples 1 and 2, the absolute value inequality was converted to an equivalent compound inequality. However, sometimes students have difficulty setting up the appropriate compound inequality. To avoid this problem, you may want to use the test point method to solve absolute value inequalities.

**PROCEDURE Solving Inequalities by Using the Test Point Method**

**Step 1** Find the boundary points of the inequality. (Boundary points are the real solutions to the related equation and points where the inequality is undefined.)

**Step 2** Plot the boundary points on the number line. This divides the number line into intervals.

**Step 3** Select a test point from each interval and substitute it into the original inequality.

- If a test point makes the original inequality true, then that interval is part of the solution set.

**Step 4** Test the boundary points in the original inequality.

- If the original inequality is strict ( or ), do not include the boundary points in the solution set.
- If the original inequality is defined using or , then include the boundary points that are well defined within the inequality.

*Note:* Any boundary point that makes an expression within the inequality undefined must *always* be excluded from the solution set.

To demonstrate the use of the test point method, we will repeat the absolute value inequalities from Examples 1 and 2. Notice that regardless of the method used, the absolute value is always isolated first before any further action is taken.

**Example 5** Solving an Absolute Value Inequality by the Test Point Method

Solve the inequality by using the test point method. \( |3w + 1| - 4 < 7 \)

**Solution:**

\[
|3w + 1| - 4 < 7 \\
|3w + 1| < 11 \\
|3w + 1| = 11
\]

\[
3w + 1 = 11 \quad \text{or} \quad 3w + 1 = -11
\]

\[
3w = 10 \quad \text{or} \quad 3w = -12
\]

\[
w = \frac{10}{3} \quad \text{or} \quad w = -4
\]

**Isolate the absolute value.**

**Step 1:** Solve the related equation.

Write as two equations.

These are the only boundary points.

**Step 2:** Plot the boundary points.
Chapter 1  Linear Equations and Inequalities in One Variable

Step 4: Because the original inequality is a strict inequality, the boundary points (where equality occurs) are not included.

The solution is \( w \neq -4 \) or, equivalently in interval notation, \((-4, \frac{10}{3})\).

Skill Practice  Solve the inequality.

7. \( 6 + |3t - 4| \leq 10 \)

Example 6  Solving an Absolute Value Inequality by the Test Point Method

Solve the inequality by using the test point method. \( 3 \leq 1 + \left| \frac{1}{2} t - 5 \right| \)

Solution:

\[
\begin{align*}
3 & \leq 1 + \left| \frac{1}{2} t - 5 \right| \\
1 + \left| \frac{1}{2} t - 5 \right| & \geq 3 \\
\left| \frac{1}{2} t - 5 \right| & \geq 2 \\
\frac{1}{2} t - 5 & = 2 \\
\frac{1}{2} t - 5 & = 2 \quad \text{or} \quad \frac{1}{2} t - 5 = -2 \\
\frac{1}{2} t & = 7 \quad \text{or} \quad \frac{1}{2} t = 3 \\
t & = 14 \quad \text{or} \quad t = 6
\end{align*}
\]

Step 1: Solve the related equation. Write as two equations. These are the boundary points.

Step 2: Plot the boundary points.

Step 3: Select a test point from each interval.

\[
\begin{array}{ccc}
\text{Interval I} & \text{Interval II} & \text{Interval III} \\
6 & 14 & \\
\end{array}
\]

Answer

7. \( \left[ \frac{8}{3}, 14 \right] \)
Test $t = 0$:  
\[3 \leq 1 + \left\{ \frac{1}{2}(0) - 5 \right\} \quad 3 \leq 1 + \left\{ \frac{1}{2}(10) - 5 \right\} \quad 3 \leq 1 + \left\{ \frac{1}{2}(16) - 5 \right\} \]
\[3 \leq 1 + |0 - 5| \quad 3 \leq 1 + |5 - 5| \quad 3 \leq 1 + |8 - 5| \]
\[3 \leq 1 + |-5| \quad 3 \leq 1 + |0| \quad 3 \leq 1 + |3| \]
\[3 \leq 6 \text{ True} \quad 3 \leq 1 \text { False} \quad 3 \leq 4 \text{ True} \]

**Step 4:** The original inequality uses the sign $\geq$. Therefore, the boundary points (where equality occurs) must be part of the solution set.

The solution is \( \{ t \mid t \leq 6 \text{ or } t \geq 14 \} \) or, equivalently in interval notation, \((-\infty, 6] \cup [14, \infty)\).

**Skill Practice** Solve the inequality.
8. \( \frac{1}{2}c + 4 + 1 > 6 \)

3. **Translating to an Absolute Value Expression**

Absolute value expressions can be used to describe distances. The distance between \( c \) and \( d \) is given by \( |c - d| \). For example, the distance between \(-2\) and \(3\) on the number line is \( |(-2) - 3| = |-5| = 5 \) as expected.

**Example 7** Expressing Distances with Absolute Value

Write an absolute value inequality to represent the following phrases.

a. All real numbers \( x \), whose distance from zero is greater than 5 units

b. All real numbers \( x \), whose distance from \(-7\) is less than 3 units

**Solution:**

a. All real numbers \( x \), whose distance from zero is greater than 5 units
\[|x - 0| > 5 \text{ or simply } |x| > 5\]

b. All real numbers \( x \), whose distance from \(-7\) is less than 3 units
\[|x - (-7)| < 3 \text{ or simply } |x + 7| < 3\]

**Skill Practice** Write an absolute value inequality to represent the following phrases.

9. All real numbers whose distance from zero is greater than 10 units
10. All real numbers whose distance from 4 is less than 6 units

**Answers**
8. \((-\infty, -18) \cup (2, \infty)\)
9. \(|x| > 10\)
10. \(|x - 4| < 6\)
Chapter 1  Linear Equations and Inequalities in One Variable

Absolutely value expressions can also be used to describe boundaries for measurement error.

**Example 8  Expressing Measurement Error with Absolute Value**

Latoya measured a certain compound on a scale in the chemistry lab at school. She measured 8 g of the compound, but the scale is only accurate to ± 0.1 g. Write an absolute value inequality to express an interval for the true mass, \( x \), of the compound she measured.

**Solution:**

Because the scale is only accurate to ± 0.1 g, the true mass, \( x \), of the compound may deviate by as much as 0.1 g above or below 8 g. This may be expressed as an absolute value inequality:

\[ |x - 8.0| \leq 0.1 \quad \text{or equivalently} \quad 7.9 \leq x \leq 8.1 \]

**Skill Practice**

11. Vonzell molded a piece of metal in her machine shop. She measured the thickness at 12 mm. Her machine is accurate to ±0.05 mm. Write an absolute value inequality to express an interval for the true measurement of the thickness, \( t \), of the metal.

**Section 1.7  Practice Exercises**

**Study Skills Exercises**

1. When you take a test, go through the test and do all the problems that you know first. Then go back and work on the problems that were more difficult. Give yourself a time limit for how much time you spend on each problem (maybe 3 to 5 min the first time through). Circle the importance of each statement.

<table>
<thead>
<tr>
<th></th>
<th>Not important</th>
<th>Somewhat important</th>
<th>Very important</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Read through the entire test first.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>b. If time allows, go back and check each problem.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>c. Write out all steps instead of doing the work in your head.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

2. Define the key term **absolute value inequality**.

**Review Exercises**

For Exercises 3–4, solve the equations.

3. \( 2 = |5 - 7x| + 1 \)

4. \( |3x - 12| + 4 = 6 - 2 \)
Section 1.7 Absolute Value Inequalities

For Exercises 5–8, solve the inequalities and graph the solution set. Write the solution in interval notation.

5. \(-15 < 3w - 6 \leq -9\)
6. \(5 - 2y \leq 1\) and \(3y + 2 \geq 14\)
7. \(m - 7 \leq -5\) or \(m - 7 \geq -10\)
8. \(3b - 2 < 7\) or \(b - 2 > 4\)

Concepts 1 and 2: Solving Absolute Value Inequalities

For Exercises 9–20, solve the equations and inequalities. For each inequality, graph the solution set and express the solution in interval notation. (See Examples 1–6.)

9. a. \(|x| = 5\)
   b. \(|x| > 5\)
   c. \(|x| < 5\)
10. a. \(|a| = 4\)
    b. \(|a| > 4\)
    c. \(|a| < 4\)
11. a. \(|x - 3| = 7\)
    b. \(|x - 3| > 7\)
    c. \(|x - 3| < 7\)
12. a. \(|w + 2| = 6\)
    b. \(|w + 2| > 6\)
    c. \(|w + 2| < 6\)
13. a. \(|p| = -2\)
    b. \(|p| > -2\)
    c. \(|p| < -2\)
14. a. \(|x| = -14\)
    b. \(|x| > -14\)
    c. \(|x| < -14\)
15. a. \(|y + 1| = -6\)
    b. \(|y + 1| > -6\)
    c. \(|y + 1| < -6\)
16. a. \(|z - 4| = -3\)
    b. \(|z - 4| > -3\)
    c. \(|z - 4| < -3\)
17. a. \(|x| = 0\)
    b. \(|x| > 0\)
    c. \(|x| < 0\)
18. a. \(|p + 3| = 0\)
    b. \(|p + 3| > 0\)
    c. \(|p + 3| < 0\)
19. a. \(|k - 7| = 0\)
    b. \(|k - 7| > 0\)
    c. \(|k - 7| < 0\)
20. a. \(|2x + 4| + 3 = 2\)
    b. \(|2x + 4| + 3 > 2\)
    c. \(|2x + 4| + 3 < 2\)

For Exercises 21–50, solve the absolute value inequalities. Graph the solution set and write the solution in interval notation. (See Examples 1–6.)

21. \(|x| > 6\)
22. \(|x| \leq 6\)
23. \(|t| \leq 3\)
24. \(|p| > 3\)
25. \(|y + 2| \geq 0\)
26. \(0 \leq |7n + 2|\)
27. \(5 \leq |2x - 1|\)
28. \(7 \leq |x - 2|\)
29. \(|k - 7| < -3\)
30. \(|h + 2| < -9\)
31. \(\left| \frac{w - 2}{3} \right| - 3 \leq 1\)
32. \(\left| \frac{x + 3}{2} \right| - 2 \geq 4\)
Chapter 1  Linear Equations and Inequalities in One Variable

Concept 3: Translating to an Absolute Value Expression
For Exercises 51–54, write an absolute value inequality equivalent to the expression given. (See Example 7.)

51. All real numbers whose distance from 0 is greater than 7
52. All real numbers whose distance from −3 is less than 4
53. All real numbers whose distance from 2 is at most 13
54. All real numbers whose distance from 0 is at least 6
55. A 32-oz jug of orange juice may not contain exactly 32 oz of juice. The possibility of measurement error exists when the jug is filled in the factory. If the maximum measurement error is ±0.05 oz, write an absolute value inequality representing the range of volumes, \( x \), in which the orange juice jug may be filled. (See Example 8.)
56. The length of a board is measured to be 32.3 in. The maximum measurement error is ±0.2 in. Write an absolute value inequality that represents the range for the length of the board, \( x \).
57. A bag of potato chips states that its weight is \( 6 \frac{1}{2} \) oz. The maximum measurement error is ±\( \frac{1}{4} \) oz. Write an absolute value inequality that represents the range for the weight, \( x \), of the bag of chips.
58. A \( \frac{5}{4} \)-in. bolt varies in length by at most ±\( \frac{1}{8} \) in. Write an absolute value inequality that represents the range for the length, \( x \), of the bolt.
59. The width, \( w \), of a bolt is supposed to be 2 cm but may have a 0.01-cm margin of error. Solve \( |w - 2| \leq 0.01 \), and interpret the solution to the inequality in the context of this problem.
60. In a poll by the Washington Post prior to the 2008 presidential election, Senator Barak Obama was projected to receive 53% of the votes with a margin of error of 3%. Solve \( |p - 0.53| \leq 0.03 \), and interpret the solution in the context of this problem.
Expanding Your Skills
For Exercises 61–64, match the graph with the inequality.

61. 62. 63. 64.

\[ |x - 2| < 4 \quad |x - 1| > 4 \quad |x - 3| < 2 \quad |x - 5| > 1 \]

Problem Recognition Exercises

Identifying Equations and Inequalities

For Exercises 1–20,

a. Identify the type of equation or inequality. Choose from:
   - linear equation
   - absolute value equation
   - linear inequality
   - compound inequality
   - absolute value inequality

b. Solve the equation or inequality. Express the solution set in interval notation where appropriate.

1. \(-0.5y + 0.7 = 3.7\)
2. \(3m - 9 = 18\)
3. \(|2r + 8| \leq 4\)
4. \(|1 - 3x| < -1\)
5. \(-11 < 2t + 1 < 19\)
6. \(2z - 3 \geq 11 \text{ or } 3z + 3 < 9\)
7. \(\left| \frac{1}{2}y + 3 \right| = 5\)
8. \(|4x + 3| = |9 - 2x|\)
9. \(\frac{3}{4}p \geq -9\)
10. \(8w + 4 \geq 5w + 1\)
11. \(\left| \frac{2x - 9}{3} \right| \geq 5\)
12. \(\left| \frac{10 - x}{5} \right| < 3\)
13. \(|2 - c| + 5 = 3\)
14. \(|10n + 2| + 7 = 7\)
15. \(\frac{w - 4}{5} - \frac{w + 1}{3} = 1\)
16. \(\frac{1}{3}y - \frac{5}{6} = \frac{1}{2}y + 1\)
17. \(2x - 7 > 9 \text{ and } 3x \leq 36\)
18. \(-3 + x > 2x \text{ and } 2 \geq -\frac{1}{3}x\)
19. \(5(x - 2) + 7 = 2x + 3(x - 1)\)
20. \(7y - 4 = 3(y + 1) + 4y\)

For Exercises 21–28, solve each equation or inequality. Express the solution in interval notation where appropriate.

21. a. \(3x - 9 = 18\)
   b. \(|3x - 9| = 18\)
   c. \(|3x - 9| < 18\)
   d. \(|3x - 9| > 18\)
22. a. \(5y + 2 = -20\)
   b. \(|5y + 2| = -20\)
   c. \(|5y + 2| \leq -20\)
   d. \(|5y + 2| > -20\)
Chapter 1  Linear Equations and Inequalities in One Variable

23. a. \(-2t - 14 = 0\)
   b. \(-2t - 14 > 0\)
   c. \(-2t - 14 \leq 0\)

24. a. \(\frac{x - 2}{3} = 9\)
   b. \(\frac{x - 2}{3} \geq 9\)
   c. \(\frac{x - 2}{3} < 9\)

25. a. \(|8t - 2| = |-2t + 3|\)
   b. \(8t - 2 = -2t + 3\)

26. a. \(-5 < x + 2\) and \(x + 2 \leq 8\)
   b. \(-5 < x + 2 \leq 8\)

27. a. \(-4x - 9 < 11\) or \(2 \leq x + 1\)
   b. \(-4x - 9 < 11\) and \(2 \leq x + 1\)

28. a. \(4 < 2y\) or \(-3(y + 2) > -2y + 1\)
   b. \(4 < 2y\) and \(-3(y + 2) > -2y + 1\)

Understanding the Symbolism of Mathematics

Estimated time: 15 minutes

Group Size: 3

As you advance in your study of mathematics, you will notice that mathematics has its own language and syntax. For example, the statement \(a > 0\) translates to “\(a\) is greater than zero.” This also means that \(a\) is positive. Read and interpret the following conditions imposed on the variables \(a, b, c, d,\) and \(x\). Then determine whether the statements in Exercises 1–12 are true or false.

\[
\begin{align*}
  a &> 0 & b &< 0 & -1 &< c &< 1 & d &> 1 & x & = 0 \\
1. \ ab &> 0 \quad \text{_____} & 2. \ bd &< 0 \quad \text{_____} & 3. \ a &+ d &> 0 \quad \text{_____} \\
4. \ b^2 &< 0 \quad \text{_____} & 5. \ a^2 &> 0 \quad \text{_____} & 6. \ c &+ d &\geq 0 \quad \text{_____} \\
7. \ c^2 &< 1 \quad \text{_____} & 8. \ d^2 &> 1 \quad \text{_____} & 9. \ |c| &< 1 \quad \text{_____} \\
10. \ bx &< 0 \quad \text{_____} & 11. \ b &+ x &> 0 \quad \text{_____} & 12. \ \frac{x}{d} & = 0 \quad \text{_____} \\
\end{align*}
\]

For Exercises 13–24, suppose that \(m\) represents an odd integer and \(n\) represents an even integer. Determine whether the statements are true or false.

13. \(m + 1\) is an odd integer. \quad \text{_____} \quad 14. \(m + 2\) is an odd integer. \quad \text{_____}
15. \(m - 1\) is an odd integer. \quad \text{_____} \quad 16. \(m - 2\) is an odd integer. \quad \text{_____}
17. \(m + n\) is an even integer. \quad \text{_____} \quad 18. \(m - n\) is an even integer. \quad \text{_____}
19. \(m^2\) is an even integer. \quad \text{_____} \quad 20. \(n^2\) is an even integer. \quad \text{_____}
21. \((m + n)^2\) is an odd integer. \quad \text{_____} \quad 22. \((n + n)^2\) is an even integer. \quad \text{_____}
23. \((m + n + 1)^3\) is an odd integer. \quad \text{_____} \quad 24. \((m + m - 2)^3\) is an odd integer. \quad \text{_____}
Chapter 1 Summary

Section 1.1 Linear Equations in One Variable

Key Concepts

A linear equation in one variable can be written in the form \( ax + b = 0 \) \((a \neq 0)\).

Steps to Solve a Linear Equation in One Variable

1. Simplify both sides of the equation.
   - Clear parentheses.
   - Consider clearing fractions or decimals (if any are present) by multiplying both sides of the equation by a common denominator of all terms.
   - Combine like terms.
2. Use the addition or subtraction property of equality to collect the variable terms on one side of the equation.
3. Use the addition or subtraction property of equality to collect the constant terms on the other side.
4. Use the multiplication or division property of equality to make the coefficient on the variable term equal to 1.
5. Check your answer and write the solution set.

An equation that has no solution is called a contradiction.

An equation that has all real numbers as its solutions is called an identity.

Examples

Example 1

\[
\frac{1}{2}(x - 4) - \frac{3}{4}(x + 2) = \frac{1}{4}
\]

\[
\frac{1}{2}x - 2 - \frac{3}{4}x - \frac{3}{2} = \frac{1}{4}
\]

\[
4\left(\frac{1}{2}x - 2 - \frac{3}{4}x - \frac{3}{2}\right) = 4\left(\frac{1}{4}\right)
\]

\[
2x - 8 - 3x - 6 = 1
\]

\[
-x - 14 = 1
\]

\[
-x = 15
\]

\[
x = -15
\]

The solution \(-15\) checks in the original equation.

The solution set is \{\(-15\)\}.

Example 2

\[
3x + 6 = 3(x - 5)
\]

\[
3x + 6 = 3x - 15
\]

6 = \(-15\) Contradiction

There is no solution, \{ \}.

Example 3

\[
-(5x + 12) - 3 = 5(-x - 3)
\]

\[
-5x - 12 - 3 = -5x - 15
\]

\[
-5x - 15 = -5x - 15
\]

\[
-15 = -15\quad \text{Identity}
\]

All real numbers are solutions.

The solution set is \{ \(x \mid x \) is a real number \}.
Section 1.2  Applications of Linear Equations in One Variable

Key Concepts

Problem-Solving Steps for Word Problems
1. Read the problem carefully.
2. Assign labels to unknown quantities.
3. Develop a verbal model.
4. Write a mathematical equation.
5. Solve the equation.
6. Interpret the results and write the final answer in words.

Sales tax: \((\text{cost of merchandise})(\text{tax rate})\)
Commission: \((\text{dollars in sales})(\text{rate})\)
Simple interest: \(I = Prt\)
Distance: \((\text{rate})(\text{time})\)  \(d = rt\)

Examples

Example 1

1. Estella needs to borrow $8500. She borrows part of the money from a friend and agrees to pay the friend 6% simple interest. She borrows the rest of the money from a bank that charges 10% simple interest. If she pays back the money at the end of 1 yr and also pays $750 in interest, find the amount that Estella borrowed from each source.

2. Let \(x\) represent the amount borrowed at 6%. Then \(8500 - x\) is the amount borrowed at 10%.

<table>
<thead>
<tr>
<th></th>
<th>6% Account</th>
<th>10% Account</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>(x)</td>
<td>(8500 - x)</td>
<td>8500</td>
</tr>
<tr>
<td>Interest</td>
<td>0.06(x)</td>
<td>0.10(8500 - (x))</td>
<td>750</td>
</tr>
</tbody>
</table>

3. \(\left(\text{Interest owed at 6%}\right) + \left(\text{interest owed at 10%}\right) = \left(\text{total interest}\right)\)

4. \(0.06x + 0.10(8500 - x) = 750\)

5. \(6x + 10(8500 - x) = 75,000\)
   \(6x + 85,000 - 10x = 75,000\)
   \(-4x = -10,000\)
   \(x = 2500\)

6. \(x = 2500\)
   \(8500 - 2500 = 6000\)

$2500 was borrowed at 6% and $6000 was borrowed at 10%.
Section 1.3  Applications to Geometry and Literal Equations

Key Concepts
Some useful formulas for word problems:

**Perimeter**
Rectangle: $P = 2l + 2w$

**Area**
Rectangle: $A = lw$
Square: $A = s^2$
Triangle: $A = \frac{1}{2}bh$
Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

**Angles**
Two angles whose measures total $90^\circ$ are complementary angles.

Two angles whose measures total $180^\circ$ are supplementary angles.

The sum of the measures of the angles of a triangle is $180^\circ$.

Literal equations (or formulas) are equations with several variables. To solve for a specific variable, follow the steps to solve a linear equation.

Examples

**Example 1**
A border of marigolds is to enclose a rectangular flower garden. If the length is twice the width and the perimeter is 25.5 ft, what are the dimensions of the garden?

The width is 4.25 ft, and the length is $2(4.25)$ ft or 8.5 ft.

**Example 2**
Solve for $y$.

$4x - 5y = 20$

$-5y = -4x + 20$

$-\frac{5y}{-5} = \frac{-4x + 20}{-5}$

$y = \frac{-4x + 20}{-5}$ or $y = \frac{4}{5}x - 4$
### Section 1.4 Linear Inequalities in One Variable

**Key Concepts**

A **linear inequality** in one variable can be written in the form

\[ ax + b < 0, \quad ax + b > 0, \quad ax + b \leq 0, \quad \text{or} \quad ax + b \geq 0 \]

**Properties of Inequalities**

1. If \( a < b \), then \( a + c < b + c \).
2. If \( a < b \), then \( a - c < b - c \).
3. If \( c \) is positive and \( a < b \), then \( ac < bc \) and \( \frac{a}{c} < \frac{b}{c} \).
4. If \( c \) is negative and \( a < b \), then \( ac > bc \) and \( \frac{a}{c} > \frac{b}{c} \).

Properties 3 and 4 indicate that if we multiply or divide an inequality by a negative value, the direction of the inequality sign must be reversed.

**Examples**

**Example 1**

Solve.

\[
\begin{align*}
\frac{14 - x}{-2} &< -3x \\
-2(\frac{14 - x}{-2}) &> -2(-3x) \quad \text{(Reverse the inequality sign.)}
\end{align*}
\]

\[
\begin{align*}
14 - x &> 6x \\
-7x &> -14 \\
-7 &< -14 \quad \text{(Reverse the inequality sign.)}
\end{align*}
\]

\[ x < 2 \]

Interval notation: \((-\infty, 2)\)

---

### Section 1.5 Compound Inequalities

**Key Concepts**

\( A \cup B \) is the **union** of \( A \) and \( B \). This is the set of elements that belong to set \( A \) or set \( B \) or both sets \( A \) and \( B \).

\( A \cap B \) is the **intersection** of \( A \) and \( B \). This is the set of elements common to both \( A \) and \( B \).

**Examples**

**Example 1**

**Union**

\[ A \cup B \]

**Intersection**

\[ A \cap B \]
• Solve two or more inequalities joined by and by finding the intersection of their solution sets.
• Solve two or more inequalities joined by or by finding the union of their solution sets.

**Example 2**

\[-7x + 3 \geq -11 \quad \text{and} \quad 1 - x < 4.5\]

\[-7x \geq -14 \quad \text{and} \quad -x < 3.5\]

\[-x \leq 2 \quad \text{and} \quad x > -3.5\]

The solution is \(\{x \mid -3.5 < x \leq 2\}\) or equivalently \((-3.5, 2]\).

**Inequalities of the form \(a < x < b\):**
The inequality \(a < x < b\) is represented by \(a < x < b\) or, in interval notation, \((a, b)\).

**Example 3**

\[5y + 1 \geq 6 \quad \text{or} \quad 2y - 5 \leq -11\]

\[5y \geq 5 \quad \text{or} \quad 2y \leq -6\]

\[y \geq 1 \quad \text{or} \quad y \leq -3\]

The solution is \(\{y \mid y \geq 1 \text{ or } y \leq -3\}\) or equivalently \((-\infty, -3] \cup [1, \infty)\).

**Example 4**

Solve.

\[-13 \leq 3x - 1 < 5\]

\[-13 + 1 \leq 3x - 1 + 1 < 5 + 1\]

\[-12 \leq 3x < 6\]

\[-12 \leq \frac{3x}{3} < \frac{6}{3}\]

\[-4 \leq x < 2\]

Interval notation: \([-4, 2)\)
**Section 1.6 Absolute Value Equations**

**Key Concepts**
The equation $|x| = a$ is an absolute value equation. For $a \geq 0$, the solution to the equation $|x| = a$ is $x = a$ or $x = -a$.

**Steps to Solve an Absolute Value Equation**
1. Isolate the absolute value to write the equation in the form $|x| = a$.
2. If $a < 0$, there is no solution.
3. Otherwise, if $a \geq 0$, rewrite the equation $|x| = a$ as $x = a$ or $x = -a$.
4. Solve the equations from step 3.
5. Check answers in the original equation.

The equation $|x| = |y|$ implies $x = y$ or $x = -y$.

**Examples**

**Example 1**
$|2x - 3| + 5 = 10$

$|2x - 3| = 5$ Isolate the absolute value.

$2x - 3 = 5$ or $2x - 3 = -5$

$2x = 8$ or $2x = -2$

$x = 4$ or $x = -1$

The solution set is $\{4, -1\}$.

**Example 2**
$|x + 2| + 5 = 1$

$|x + 2| = -4$ No solution, $\{\}$

**Example 3**
$|2x - 1| = |x + 4|$

$2x - 1 = x + 4$ or $2x - 1 = -(x + 4)$

$x = 5$ or $2x - 1 = -x - 4$

or $3x = -3$

or $x = -1$

The solution set is $\{5, -1\}$.

---

**Section 1.7 Absolute Value Inequalities**

**Key Concepts**

**Solutions to Absolute Value Inequalities**

For $a > 0$, we have:

$|x| > a \Rightarrow x < -a \text{ or } x > a$

$|x| < a \Rightarrow -a < x < a$

**Examples**

**Example 1**
$|5x - 2| < 12$

$-12 < 5x - 2 < 12$

$-10 < 5x < 14$

$-2 < x < \frac{14}{5}$

The solution is $\left(-2, \frac{14}{5}\right)$. 
Test Point Method to Solve Inequalities

1. Find the boundary points of the inequality. (Boundary points are the real solutions to the related equation and points where the inequality is undefined.)
2. Plot the boundary points on the number line. This divides the number line into intervals.
3. Select a test point from each interval and substitute it into the original inequality.
   • If a test point makes the original inequality true, then that interval is part of the solution set.
4. Test the boundary points in the original inequality.
   • If the original inequality is strict (< or >), do not include the boundary in the solution set.
   • If the original inequality is defined using ≤ or ≥, then include the boundary points that are well defined within the inequality.

Note: Any boundary point that makes an expression within the inequality undefined must always be excluded from the solution set.

If a is negative (a < 0), then

1. |x| < a has no solution.
2. |x| > a is true for all real numbers.

Example 2

\[ |x - 3| + 2 \geq 7 \]
\[ |x - 3| \geq 5 \quad \text{Isolate the absolute value.} \]
\[ |x - 3| = 5 \quad \text{Solve the related equation.} \]
\[ x - 3 = 5 \quad \text{or} \quad x - 3 = -5 \]
\[ x = 8 \quad \text{or} \quad x = -2 \quad \text{Boundary points} \]

\[
\begin{array}{ccc}
\text{Interval I:} & \text{Interval II:} & \text{Interval III:} \\
-2 & 2 & 8
\end{array}
\]

**Interval I:**
Test \( x = -3: \) \[ |(-3) - 3| + 2 \geq 7 \] False

**Interval II:**
Test \( x = 0: \) \[ |(0) - 3| + 2 \geq 7 \] False

**Interval III:**
Test \( x = 9: \) \[ |(9) - 3| + 2 \geq 7 \] True

The solution is \((-\infty, -2] \cup [8, \infty).\)

Example 3

\[ |x + 5| > -2 \]

The solution is all real numbers because an absolute value will always be greater than a negative number.

\((-\infty, \infty)\)

Example 4

\[ |x + 5| < -2 \]

There is no solution because an absolute value cannot be less than a negative number.

The solution set is \(\{\}\).
Chapter 1 Review Exercises

Section 1.1

1. Describe the solution set for a contradiction.
2. Describe the solution set for an identity.

For Exercises 3–12, solve the equations and identify each as a conditional equation, a contradiction, or an identity.

3. \( x - 27 = -32 \)
4. \( y + \frac{7}{8} = 1 \)
5. \( 7.23 + 0.6x = 0.2x \)
6. \( 0.1y + 1.122 = 5.2y \)
7. \( -(4 + 3m) = 9(3 - m) \)
8. \( -2(5n - 6) = 3(-n - 3) \)
9. \( \frac{x - 3}{5} - \frac{2x + 1}{2} = 1 \)
10. \( 3(x + 3) - 2 = 3x + 2 \)
11. \( \frac{10}{8}m + 18 - \frac{7}{8}m = \frac{3}{8}m + 25 \)
12. \( \frac{2}{3}m + \frac{1}{3}(m - 1) = -\frac{1}{3}m + \frac{1}{3}(4m - 1) \)

Section 1.2

13. Explain how you would label three consecutive integers.
14. Explain how you would label two consecutive odd integers.
15. Explain what the formula \( d = rt \) means.
16. Explain what the formula \( I = Prt \) means.
17. a. Cory made $30,403 in taxable income. If he pays 28% in federal income tax, determine the amount of tax he must pay.
   b. What is his net income (after taxes)?
18. For a recent year, approximately 7.2 million men were in college in the United States. This represents an 8% increase over the number of men in college in 2000. Approximately how many men were in college in 2000? (Round to the nearest tenth of a million.)

Section 1.3

25. The length of a rectangle is 2 ft more than the width. Find the dimensions if the perimeter is 40 ft.

For Exercises 26–27, solve for \( x \), and then find the measure of each angle.

26. \( (x - 25)^{\circ} = \left(\frac{x}{2} + 1\right)^{\circ} \)
For Exercises 28–31, solve for the indicated variable.

28. $3x - 2y = 4$ for $y$
29. $-6x + y = 12$ for $y$
30. $S = 2\pi r + \pi r^2h$ for $h$
31. $A = \frac{1}{2}bh$ for $b$

32. a. The circumference of a circle is given by $C = 2\pi r$. Solve this equation for $\pi$.

b. Tom measures the radius of a circle to be 6 cm and the circumference to be 37.7 cm. Use these values to approximate $\pi$. (Round to 2 decimal places.)

Section 1.4

For Exercises 33–38, solve the inequality. Graph the solution and write the solution set in interval notation.

33. $-6x - 2 \geq 6$
34. $-10x \leq 15$
35. $5 - 7(x + 3) > 19x$
36. $4 - 3x \geq 10(-x + 5)$
37. $\frac{5 - 4x}{8} \geq 9$
38. $\frac{3 + 2x}{4} \leq 8$

Section 1.5

39. Explain the difference between the union and intersection of two sets. You may use the sets $C$ and $D$ in the following diagram to provide an example.

Let $X = \{x \mid x \geq -10\}$, $Y = \{x \mid x < 1\}$, $Z = \{x \mid x > -1\}$, and $W = \{x \mid x \leq -3\}$. For Exercises 40–45, find the intersection or union of the sets $X$, $Y$, $Z$, and $W$. Write the answers in interval notation.

40. $X \cap Y$
41. $X \cup Y$
42. $Y \cap Z$
43. $Y \cap Z$
44. $Z \cup W$
45. $Z \cap W$

For Exercises 46–55, solve the compound inequalities. Write the solutions in interval notation.

46. $4m > -11$ and $4m - 3 \leq 13$
47. $4n - 7 < 1$ and $7 + 3n \geq -8$
48. $-3y + 1 \geq 10$ and $-2y - 5 \leq -15$
49. $\frac{1}{2} - \frac{h}{12} \leq -\frac{7}{12}$ and $\frac{1}{2} - \frac{h}{10} > \frac{1}{5}$
50. $\frac{2}{3}t - 3 \leq 1$ or $\frac{3}{4}t - 2 > 7$
51. $2(3x + 1) < -10$ or $3(2x - 4) \geq 0$
52. $-7 < -7(2w + 3)$ or $-2 < -4(3w - 1)$
53. $(p + 3) + 4 > p - 1$ or $(p - 1) + 2 > p + 8$
54. $2 \geq -(b - 2) - 5b \geq -6$
55. $-4 \leq \frac{1}{2}(x - 1) < -\frac{3}{2}$
65. The product of $\frac{1}{2}$ and the sum of a number and 3 is between $-1$ and 5. Find all such numbers.

66. $16 = |x + 2| + 9$

67. $|4x - 1| + 6 = 4$

68. $|3x - 1| + 7 = 3$

69. $\left| \frac{7x - 3}{5} \right| + 4 = 4$

70. $\left| \frac{4x + 5}{-2} \right| - 3 = -3$

71. $|3x - 5| = |2x + 1|$

72. $|8x + 9| = |8x - 1|$

73. Which absolute value expression represents the distance between 3 and $-2$ on the number line?

\[ |3 - (-2)| \quad |-2 - 3| \]

### Section 1.7

74. Write the compound inequality $x < -5$ or $x > 5$ as an absolute value inequality.

75. Write the compound inequality $-4 < x < 4$ as an absolute value inequality.

For Exercises 76–77, write an absolute value inequality that represents the solution sets graphed here.

76. 

77. 

For Exercises 78–91, solve the absolute value inequalities. Graph the solution set and write the solution in interval notation.

78. $|x + 6| \geq 8$

79. $|x + 8| \leq 3$

80. $2|7x - 1| + 4 > 4$

81. $4|5x + 1| - 3 > -3$

82. $|3x + 4| - 6 \leq -4$

83. $|5x - 3| + 3 \leq 6$

84. $\frac{|x|}{2} - 6 < 5$

85. $\left| \frac{x}{3} + 2 \right| < 2$

86. $|4 - 2x| + 8 \geq 8$
Chapter 1 Test

For Exercises 1–4, solve the equations.

1. \( \frac{x}{7} + 1 = 20 \)
2. \( 8 - 5(4 - 3z) = 2(4 - z) - 8z \)
3. \( 0.12(x) + 0.08(60,000 - x) = 10,500 \)
4. \( \frac{5 - x}{6} - \frac{2x - 3}{2} = \frac{x}{3} \)

5. Label each equation as a conditional equation, an identity, or a contradiction.
   a. \( (5x - 9) + 19 = 5(x + 2) \)
   b. \( 2a - 2(1 + a) = 5 \)
   c. \( (4w - 3) + 4 = 3(5 - w) \)

6. The difference between two numbers is 72. If the larger is 5 times the smaller, find the two numbers.

7. Joëlle is determined to get some exercise and walks to the store at a brisk rate of 4.5 mph. She meets her friend Yun Ling at the store, and together they walk back at a slower rate of 3 mph. Joëlle’s total walking time was 1 hr.
   a. How long did it take her to walk to the store?
   b. What is the distance to the store?

8. Shawna banks at a credit union. Her money is distributed between two accounts: a certificate of deposit (CD) that earns 5% simple interest and a savings account that earns 3.5% simple interest. Shawna has $100 less in her savings account than in the CD. If after 1 year her total interest is $81.50, how much did she invest in the CD?

9. A yield sign is in the shape of an equilateral triangle (all sides have equal length). Its perimeter is 81 in. Find the length of the sides.

10. The sum of three consecutive odd integers is 41 less than four times the largest. Find the numbers.

11. How many gallons of a 20% acid solution must be mixed with 6 gal of a 30% acid solution to make a 22% solution?

For Exercises 12–13, solve the equations for the indicated variable.

12. \( 4x + 2y = 6 \) for \( y \)  
13. \( x = \mu + z\sigma \) for \( z \)

For Exercises 14–16, solve the inequalities. Graph the solution and write the solution set in interval notation.

14. \( x + 8 > 42 \)  
15. \( -\frac{3}{2}x + 6 \geq x - 3 \)
16. \( -2 < 3x - 1 \leq 5 \)

94. The Nielsen ratings estimated that the percent, \( p \), of the television viewing audience watching American Idol was 20% with a 3% margin of error. Solve the inequality \( |p - 0.20| \leq 0.03 \) and interpret the answer in the context of this problem.

95. The length, \( L \), of a screw is supposed to be \( \frac{3}{2} \) in. Due to variation in the production equipment, there is a \( \frac{1}{4} \) in. margin of error. Solve the inequality \( |L - \frac{3}{2}| \leq \frac{1}{4} \) and interpret the answer in the context of this problem.
Chapter 1  Linear Equations and Inequalities in One Variable

17. An elevator can accommodate a maximum weight of 2000 lb. If four passengers on the elevator have an average weight of 180 lb each, how many additional passengers of the same average weight can the elevator carry before the maximum weight capacity is exceeded?

23. The normal range in humans of the enzyme adenosine deaminase (ADA), is between 9 and 33 IU (international units), inclusive. Let $x$ represent the ADA level in international units.
   a. Write an inequality representing the normal range for ADA.
   b. Write a compound inequality representing abnormal ranges for ADA.

For Exercises 24–26, solve the absolute value equations.

24. \[ \frac{1}{2}x + 3 = 4 \]
25. \[ |3x + 4| = |x - 12| \]
26. \[ -5 = -8 + |2y - 3| \]

For Exercises 27–31, solve the absolute value inequalities. Write the answers in interval notation.

27. \[ |3 - 2x| + 6 < 2 \]
28. \[ |3x - 8| \geq 9 \]
29. \[ |0.4x + 0.3| - 0.2 < 7 \]
30. \[ |7 - 3x| + 1 > -3 \]
31. \[ 6 \leq |2x - 5| - 5 \]
32. The mass of a small piece of metal is measured to be 15.41 g. If the measurement error is at most ±0.01 g, write an absolute value inequality that represents the possible mass, $x$, of the piece of metal.

For Exercises 18–22, solve the compound inequalities. Write the answers in interval notation.

18. \[ -2 \leq 3x - 1 \leq 5 \]
19. \[ -\frac{3}{5}x - 1 \leq 8 \quad \text{or} \quad \frac{2}{3}x \geq 16 \]
20. \[ -2x - 3 > -3 \quad \text{and} \quad x + 3 \geq 0 \]
21. \[ 5x + 1 \leq 6 \quad \text{or} \quad 2x + 4 > -6 \]
22. \[ 2x - 3 > 1 \quad \text{and} \quad x + 4 < -1 \]